VII. *A Method of finding the Latitude of a Place, by Means of two Altitudes of the Sun and the Time elapsed betwixt the Observations. By the Rev.* W. Lax, *A. M.* Lowndes's *Professor of Astronomy in the University of Cambridge.*

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I hope the following method of determining the latitude, by means of two altitudes of the sun and the time elapsed betwixt the observations, will be found not less convenient for nautical purposes than the rules which are commonly employed. But I would rather recommend it in those cases where rigid accuracy is required, and the astronomer is provided with no better instrument for taking the sun's altitude than a Hadley's sextant of the most improved construction. The process will be neither difficult nor tedious; and, if the observations are made with proper exactness, I conceive the latitude will generally be obtained within a few seconds of the truth. With these expectations, I have ventured to reduce the method into its present form; and submit it, with the utmost deference, to this learned Society.

In the spherical triangle, whose sides are the complements of the latitude, declination, and altitude, let z represent the angle at the pole, and t its tangent; Z the azimuth, and T its tangent; L the latitude, and λ its cosine, radius being unity; then, if the altitude and declination remain constant, we shall have $\dot{L} = \lambda T \dot{z}$, and, consequently, $\dot{L}$ will vary as $T \dot{z}$, when the increment of λ, compared with λ itself, is inconsiderable. Hence, if the abscisse of the curve *ABCD* (fig. 1.) be always proportional to *z*, and its ordinate to *T*, the area *GB* intercepted betwixt any two of these ordinates may represent the increment of the latitude corresponding to the increment of the time *EG*. Let *abcd* (fig. 2.) be another curve, whose abscisse *ae* is always equal to *AE* in the preceding figure, but whose ordinate *eb* is proportional to *t*, the tangent of the hour-angle; then will the area *gb* vary as *GB*, at small distances from the meridian, and, of course, may represent the increment of the latitude. Now, to prove this, we have only to shew that *T* and *t*, when both are small, bear to each other a given ratio.

FIG. 1.

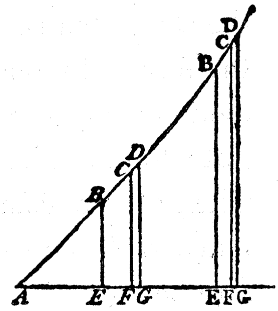
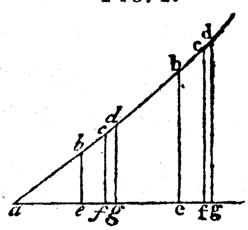


FIG. 2.



Let *S* and Σ be the sine and cosine of the azimuth; *s* and σ the sine and cosine of the angle at the pole; then will $\frac{\dot{T}}{T} = \dot{Z} . \frac{1 + T^2}{T}$, and $\frac{\dot{t}}{t} = \dot{z} . \frac{1 + t^2}{t}$; $\dot{Z} = \frac{\dot{S}}{\Sigma}$, and $\dot{z} = \frac{\dot{s}}{\sigma}$. But, since the complements of the declination and altitude remain constant, whilst the latitude is made to vary, $\dot{S}$ will be to $\dot{s}$ as *S* to *s*; and, therefore, we shall have $\frac{\dot{T}}{T} : \frac{\dot{t}}{t} :: \frac{S}{\Sigma} \times \frac{1 + T^2}{T} : \frac{s}{\sigma} \times \frac{1 + t^2}{t}$ $:: 1 + T^2 : 1 + t^2 ::$ the square of the secant of the azimuth : the square of the secant of the hour-angle, which may be considered as a ratio of equality, when the angles are very small. The fluxions, therefore, of the tangents are as the tangents themselves; and, consequently, they must always preserve the same ratio towards each other. Let us now suppose that an altitude of the sun is taken at the distance *ae* from the meridian, but that, in consequence of an error in the assumed latitude, the calculated time is *ag*; and that, with a lat. differing from the former by one minute, we compute again, and the time is found equal to *af*; then will the area *gc* be to *gb* as one minute to the whole error in latitude. Let another altitude be taken at the distance *ae* from noon, and let the times computed with the two different latitudes that were employed before be *ag* and *af*; then, in this case likewise, the area gc will be to the area gb, as one minute to the error in latitude. Now the latter curve is the "*figura tangentium,*" whose quadrature is given by COTES, in his *Harmonia Mensurarum*, and the expression for which is extremely simple. For the fluxion of the area is $= \frac{t\dot{t}}{1 + t^2}$, and the area itself $=$ log. $\overline{1 + t^2}|^{\frac{1}{2}} =$ log. secant of the angle at the pole. The difference of the log. secants, or log. cosines, will of course be equal to the area intercepted betwixt the tangents which correspond to them.

Hence a table might easily be constructed with a double argument,—the distance from noon, and the variation in time arising from the different suppositions of latitude,—which might immediately exhibit the logarithm of the area corresponding to any particular base *eg* supposed to be given. A second table might have for its argument the difference of the logarithms of the area *gb* and the area *gc*, (which is likewise conceived to be known) and discover at once, in degrees, minutes, and seconds, the correction to be made in the assumed latitude. This correction, as it appears from a comparison of the signs of $\dot{L}$ and $\dot{z}$ in the equation $\dot{L} = \lambda T \dot{z}$, must be added or subtracted, according as the distance from noon obtained by computation is too great or too little, when the azimuth is less than 90 degrees; but the contrary, when the azimuth exceeds a right angle. Tables of the above description shall be constructed, if this method be received with approbation; and, in the mean time, it is proposed to subjoin a short specimen which is already completed.

I have presumed that we are able to determine *eg*, the error in time arising from an error in the assumed latitude, at either of the observations; and hence it becomes necessary, before we can avail ourselves of the principles which have been laid down, to point out the manner in which this may be accomplished. The clock gives us the whole interval betwixt the observations (supposed to be made on different sides of the meridian) equal to *ae* + ae, and by computation we obtain *ag* + ag, and thence we deduce *eg* + eg the whole error in time. Now the area *gb* is equal to gb; and therefore, if we make a rough division of the whole error, without any regard to accuracy, in the inverse ratio of the hour-angles at the two observations, and, entering the first table with these times, mark the area corresponding to each at their respective distances from noon, and increase the one and diminish the other equally, till we get the areas of the same magnitude, this, we may conclude, is the proper value of each.

If the table were constructed to every second of time, we might ascertain these logarithmic areas merely from inspection; but, as it will be advisable to confine it within narrower limits, we shall sometimes find it necessary, as in other tables, to deduce their ultimate value by the rule of three. When we have increased one portion of time and diminished the other, till the difference of their corresponding areas becomes a *minimum*, we must divide this difference betwixt them, in the proportion of their respective increments in the next interval of time, and subtract or add the part assigned to each, according as it is greater or less than the other. The table, however, might easily be carried to such an extent, that exactness in this division could never be required; but, on the contrary, it would be quite sufficient, when the hour-angles were nearly equal, to add the areas together, and take half the sum for the value of each.

From these principles may be deduced the following practical rule for determining the latitude of a place. When the sun comes within fifteen degrees of the meridian, in the morning, let his altitude be taken, and the time of the observation be accurately marked; and let another altitude be taken after he has passed the meridian, whilst his distance from it is less than fifteen degrees; and let the time of this observation likewise be noted. Then, with the supposed latitude of the place, compute the times corresponding to each of the altitudes in terms of the log. cosi of the hour-angle, and take the difference of the intervals, shewn by the clock, and determined by calculation, and divid it betwixt the observations in the manner explained above. Compute the log. cosine of the hour-angle a second time, with the greatest altitude and the latitude increased or diminished by a minute, according as it appears, from a comparison of the intervals, to have been too little or too great; and take the difference betwixt this log. cosine and that which resulted from the first operation, when the same altitude was employed. Having thus obtained the two areas *gb* and *gc*, we must subtract their logarithms from each other, and with their difference entering the second table we shall find the degrees, minutes, and seconds, by which the assumed latitude is to be increased or diminished.

It will be needless, perhaps, to suggest that, in the higher latitudes, we may extend the limits above specified a few degrees farther from the meridian, without offering any material violence to the theory, as it has hitherto been explained; and that, on the other hand, when the declination and latitude are nearly equal and of the same denomination, it will be expedient to confine our observations within a much shorter distance from noon. But it will afterwards be demonstrated that, whatever be the magnitude of the hour-angles, or however nearly the latitude and declination may approach towards each other, we can always secure, with very little additional trouble, an exact conclusion.

We may remark that the latitude, determined in this manner, will be nearly equivalent, in point of accuracy, to the mean result of two meridian altitudes. For we know that the increment of latitude : increment of altitude : : radius : cosine of azimuth; and, since the cosine of a small angle differs so little from the radius, this may be considered, within the limits which I have prescribed, as a ratio of equality. If, therefore, one altitude of the sun were taken, and we could ascertain the error in time arising from an error in the assumed latitude without the aid of a second observation, the latitude would be discovered with nearly the same precision as if it had been deduced from the meridian altitude. But, by means of a second observation made on a different side of noon, we obtain a second error in time of the same kind; and this being added to the former, and their sum divided in a just proportion betwixt the two observations, the same effect will be produced, with respect to the accuracy of the result, as if two latitudes had been deduced from meridian altitudes, and a mean betwixt them had been taken.

I might perhaps be allowed to say more; for I am satisfied, from experience, that I can take an altitude of the sun with greater exactness, when he is in any other situation, than when he is upon the meridian. If we could ascertain, within a few seconds, or even within a minute, the time when he attains his greatest altitude, there would then be no reason why an observation should not be made with the same degree of certainty in this, as in other cases; but we are generally obliged to keep our eye stedfastly fixed, for several minutes, upon the two images, and it is well known that, in such circumstances, the best eyes are apt to be deceived. Besides, it is impossible to preserve the contact of the limbs by perpetually moving the index, whilst the sun continues to ascend so very slowly. We are compelled to wait till they are evidently separated, and then, by one turn of the screw, to bring them into contact again, which must necessarily be a source of some inaccuracy. It is for the first of these reasons that, in taking an altitude of the sun, when he is near the meridian, I have found it advisable, not, in the usual manner, to bring the images almost to touch each other, and then to wait till they actually do so, but to bring them at once into contact, with such a degree of velocity as would make them sensibly overlap, or separate, whilst the clock beats a second.

But I consider it as one of the principal advantages of this method, that we can avail ourselves of any number of altitudes, and, of course, approximate as near as we please to a true conclusion with so little additional labour. If there be an equal number of observations made on each side of the meridian, we must combine them together by pairs, according to the preceding instructions, and thus determine the different logarithmic values of gb. Having then added them all together, and taken a mean betwixt them, we have only to compute a single incremental area gc with any of the altitudes and the lat. varied one minute, and, subtracting its log. from the mean log. value of gb, we shall obtain a very accurate correction of the assumed latitude. But, if there be more observations on one side of the meridian than on the other, when all the pairs have been united, and the areas resulting from them found, we may combine the supernumerary observations on either side with any of those which are made on the opposite side. The properest, however, for this purpose, is the observation which is made at the least distance from the meridian.

I should hope, moreover, the practical astronomer will think it a circumstance of some moment, that the principal part of the work consists in finding the time, an operation which he is obliged so frequently to perform. Any of the three methods which are usually adopted upon this occasion might easily be applied to the tables which have been described; but I will venture to recommend a different rule, which I conceive to be better adapted to our purpose than any of the others, and to which the directions before given had a particular reference.

Let a be the sine of the altitude; γ the cosine of the hour-angle; d the sine, δ the cosine, and r the tangent of declination; l the sine, λ the cosine, and s the tangent of the latitude. Then $\gamma = \frac{a - dl}{\delta\lambda} = \frac{a}{\delta\lambda} - \frac{dl}{\delta\lambda} = \frac{ars}{\delta\lambda rs} - rs = \overline{\frac{a}{dl} - 1} \,.\, rs$, when radius is unity, but $= \overline{\frac{am^3}{dl} - m^2} \,.\, \frac{rs}{m^3}$, when radius is m $= \frac{rs}{m^3} \times^{9}$. into the square of the tangent of the arc whose secant is $\left.\overline{\frac{am^3}{dl}}\right|^{\frac{1}{2}}$. Hence we deduce the following rule for determining the log. cosine of the angle at the pole. From the log. sine of the altitude increased by three times the log. radius, subtract the sum of the log. sines of the latit. and declination; take half of the remainder, and, considering it as the log. secant of an arc, find the log. tangent corresponding; multiply this by 2, and add the log. tangents of the latit. and declin. and reject thrice the log. radius; the sum will be the log. cosine of the angle required. But, when the declin. and latit. are of a different denomination, it is evident that our expression becomes $\overline{m^2 + \frac{am^3}{dl}} \cdot \frac{rs}{m^3}$, which is equal to $\frac{rs}{m^3} \times^9$. into the square of the secant of the arc whose tangent is $\overline{\frac{am^3}{dl}}\Big|^{\frac{1}{2}}$. In this case, therefore, having found the log. value of $\frac{am^3}{dl}$, and divided it by 2, we must consider the quotient as the log. tangent of an arc, whose log. secant being taken, we are to proceed as in the former case.

The advantages of this rule are obvious. We obtain the angle in terms of the log. cosine; and, consequently, when we have calculated the second time with the new latit. we have only to subtract one result from the other, and we immediately determine the area corresponding to the difference of the times. Besides, in the second computation, fewer of the elements are changed by this rule, than by any of those which are usually employed; and this is a consideration of much importance. But, if we are disposed to adopt the following method of ascertaining the incremental area gc, this advantage will be found still greater. Let us resume the expression $\gamma = \frac{a - dl}{\delta\lambda}$, and we shall have $\dot{\gamma} = \frac{1}{\delta} \times \frac{-d\lambda\dot{l} - a\dot{\lambda} + dl\dot{\lambda}}{\lambda\lambda^1}$ (λ^1 be-

ing the succeeding value of $\lambda) = \frac{1}{\delta} \times \frac{\frac{d\lambda^2\dot{\lambda}}{l} - a\dot{\lambda} + dl\dot{\lambda}}{\lambda\lambda^1} =$
$\frac{d\lambda^2\dot{\lambda} - al\dot{\lambda} + dl^2\dot{\lambda}}{\delta l\lambda\lambda^1} = \frac{d\lambda\dot{\lambda}}{\delta l\lambda^1} - \frac{a-dl}{\delta\lambda} \times \frac{\dot{\lambda}}{\lambda^1} = \frac{\overline{r}}{s} - \gamma \cdot \frac{\dot{\lambda}}{\lambda^1} =$
(taking $\dot{\lambda}$ positive instead of negative, as it ought to be when l is positive) $\overline{\gamma - \frac{r}{s}} \cdot \frac{\dot{\lambda}}{\lambda^1}$; and, consequently, $\frac{\dot{\gamma}}{\gamma} = \overline{1 - \frac{r}{s\gamma}} \cdot \frac{\dot{\lambda}}{\lambda^1}$, when radius is unity, but $= \overline{m^2 - \frac{rm^3}{s\gamma}} \cdot \frac{\dot{\lambda}}{\lambda^1 m^2}$, when radius is m. Now $\frac{\dot{\gamma}}{\gamma}$ may be considered as the increment of the hyperbolic log. of γ, and therefore, with its proper modulus, may represent the area which is the object of our investigation. We may suppose the other side of the equation to be the square of the cosine of the arc whose sine is $\overline{\frac{rm^3}{s\gamma}}\Big|^{\frac{1}{2}} \times^9$. into the increment of the hyperbolic log. of λ^1, divided by the square of the radius; and if, instead of taking this log. with the hyperbolic, we take it with BRIGGS's modulus, we must then consider $\frac{\dot{\gamma}}{\gamma}$ as the increment of the log. of γ, according to the same system. But $\frac{\dot{\lambda}}{\lambda^1}$ being equal to $\frac{s\dot{L}}{m^2}$ (when $\dot{L}$ is only one minute), it will vary as s; and, therefore, if its value be determined according to BRIGGS's system, when s is equal to radius, and be denominated v, its value in any other case will be expressed by $\frac{vs}{m}$.

Hence, to obtain the log. of the area gc, the quantity with which we are immediately concerned, we must find the log. value of $\frac{rm^3}{s\gamma}$, and divide it by 2; we must then take out the log. cosine of the arc whose log. sine is equal to the quotient; and, having multiplied it by 2, we must add the product to the constant log. of v (3,1015), and the log. tangent of the supposed latitude, rejecting thrice the log. radius. But, if $\overline{\frac{r\,m^3}{s\,\gamma}}\Big|^{\frac{1}{2}}$ be greater than radius, which must necessarily be the case when the azimuth is greater than a right angle, we must then consider $\frac{r\,m^3}{s\,\gamma} - m^2$ as the square of the tangent of the arc whose secant is $\overline{\frac{r\,m^3}{s\,\gamma}}\Big|^{\frac{1}{2}}$, observing in other respects the directions before given. The quantities r and s are both employed in the first computation, from the result of which we also obtain γ; and, consequently, this operation will not be attended with much trouble.

The above instructions, it is manifest, are given upon the supposition of r and s having the same sign; but, if the declin. and latit. should not be of a similar denomination, then will our expression become $\overline{m^2 + \frac{r\,m^3}{s\,\gamma}} \cdot \frac{v\,s}{m^3}$, and we must consider $m^2 + \frac{r\,m^3}{s\,\gamma}$ as the square of the secant whose corresponding tangent is $\overline{\frac{r\,m^3}{s\,\gamma}}\Big|^{\frac{1}{2}}$. With this exception, the process will be the same as when the tangents r and s are both affirmative.

The preceding *formula* naturally suggests to us another method of finding the log. area gc; and, as some perhaps may think this more eligible than either of the former, I shall take the liberty of explaining it. When the latit. is given, the area GC, it is obvious, must invariably preserve the same magnitude at all distances from the meridian; and, consequently, the area gc, which is proportional to it, must likewise remain constant. If, therefore, we can ascertain this area when the hour-angle is supposed to vanish, we may employ it when the sun is at any distance from noon. Let us now conceive the declin. to be equal to nothing; then will our expression for the area gc become $\frac{\dot{\lambda}}{\lambda^{\prime}}$; and, consequently, (since the tangents of the azimuth and hour-angle vanish in the ratio of their sines, or of the sines of the opposite sides in the triangle alluded to before,) we shall have the area $GC = \frac{\dot{\lambda} m}{\lambda^{\prime} l}$, when the sun is upon the meridian. But this area is always the same when the latitude is given, whatever be the sun's declination, and therefore may always be represented by $\frac{\dot{\lambda} m}{\lambda^{\prime} l} = \frac{v s}{m} \times \frac{m}{l} = \frac{v m}{\lambda}$; and the area gc will be generally expressed by $\frac{v m}{\lambda} \times \frac{\text{cos. of merid. altit.}}{\text{cos. of declin.}}$, when the hour-angle does not exceed the limits which have been recommended. Hence, if we add together the constant log. 3,1015, the log. radius, and the log. cosine of the merid. altitude, and subtract from their sum the log. cosines of the latit. and declin. we shall obtain the log. value of gc.

It will be necessary, perhaps, to meet an objection which some may be inclined to urge against the method of deducing the hour-angle in terms of the cosine, when this angle is very small. But it should be recollected, that with the angle itself we have no immediate concern, the accuracy of our conclusion depending entirely upon the accuracy with which the area corresponding to any particular increment of time can be determined. Now this area, whatever be the sun's distance from the meridian, will be nearly proportional to the increment of the latit. and, consequently, its magnitude is totally unconnected with that of the hour-angle. A given error in the quantity which expresses this area will equally affect our conclusion, whether the angle be 2, or whether it be 20 degrees. But let us inquire what effect will actually be produced, by admitting an error of half an unit in each of the log. cosines whose difference is equal to the area gc; and, of course, in some instances, an error of an unit in the area itself, upon any particular supposition of latit. and declin. We have only to ascertain the ratio which this area bears to unity; for the same ratio will the correction of the latit. bear to the error in our result. If the latit. for instance, be 50°, and the declin. 10°, on the same side of the equator with the latit. then, radius being unity, $\dot{z}$ (the increment of the hour-angle) will equal $\frac{\dot{L}}{\lambda T}$ $= \frac{\dot{L}}{\cos. 50^\circ \times \frac{S}{\Sigma}} =$ (g being the sine of the hour-angle, and α the cos. of the altit.) $\frac{\dot{L}}{\cos. 50^\circ} \times \frac{\sqrt{\alpha^2 - g^2 \delta^2}}{g \delta} =$ 11 minutes nearly, when z is 5°; and we have seen that, at any other distance from the meridian, the incremental area will be of the same magnitude. Hence, subtracting the log. cosine of 5° from that of 5° 11′ 0″, we get the difference equal to 1238; and, consequently, the error in our approximation will be to the error in the assumed latit. as 1 : 1238, when the log. cosines are carried to seven places of decimals.

But, when the zenith distance of the sun, at his greatest altit. is very small, and there is moreover a considerable uncertainty with respect to the latit. this error will probably become of more importance, and we may find it necessary to guard against it. Now it is manifest that, by diminishing the multiple which the area gb is of the area gc, exactly in the same proportion we shall diminish this error; and I shall afterwards explain in what manner it may be accomplished. The same expedient is also calculated to prevent another species of inaccuracy, to which, under similar circumstances, our process is liable.

If we are desirous of knowing how much our conclusion is affected by substituting the tangent of the hour-angle for the tangent of the azimuth, it may be done with the greatest facility. We have seen, that the area *GC* (fig. 1.) is to *GB* as one minute to the error in the assumed latitude. But our method supposes that *gc* (fig. 2.) is to *gb* as one minute to the correction required; and we must therefore estimate the difference betwixt these ratios, in order to ascertain how far the approximation is inaccurate. Let $EG = m \times FG$; and then, since *GC* may be considered as $= \frac{GD + FC}{2} \times FG$, and $GB = \frac{GD + EB}{2} \times EG = \frac{GD + EB}{2} \times m \times FG$, the former ratio will equal $\frac{GD + EB}{GD + FC} \times m =$ (according to the notation employed above) $m . \frac{2\text{T} - m\dot{\text{T}}}{2\text{T} - \dot{\text{T}}} = m - m . \frac{m-1}{2} \times \frac{\dot{\text{T}}}{\text{T}}$ nearly $=$ the minutes contained in the computed error of latitude. Their difference will equal $m . \frac{m-1}{2} . \overline{\frac{\dot{\text{T}}}{\text{T}} - \frac{\dot{t}}{t}} = m . \frac{m-1}{2} . \dot{z} . \overline{\frac{1+\text{T}^2}{t} - \dot{z} . \frac{1+t^2}{t}}$ $= m . \frac{m-1}{2} . \frac{\text{T}^2 - t^2}{t} . \dot{z} = m . \frac{m-1}{2} . \frac{\text{T}^2 - t^2}{\lambda \text{T} t} . \dot{\text{L}}$ $\left(\frac{\dot{\text{L}}}{\lambda \text{T}}\right.$ being substituted for its equal $\left.\dot{z}\right) =$ the minutes contained in the error of our approximation. It will be more convenient, however, to express this difference independently of the azimuth. Now, preserving the notation before adopted, we have $\text{T} = \frac{\text{S}}{\Sigma} = \frac{g\delta}{\alpha \times \frac{l\alpha - d}{\lambda\alpha}} = \frac{t\gamma\delta\lambda}{l\alpha - d} = \frac{t\gamma\delta}{l\gamma\delta - d\lambda}$; therefore,

$$\frac{\text{T}}{t} = \frac{\gamma\delta}{l\gamma\delta - d\lambda}, \text{ and } \frac{\text{T}}{t} - \frac{t}{\text{T}} \left(= \frac{\text{T}^2 - t^2}{\text{T}t}\right) = \frac{\gamma\delta}{l\gamma\delta - d\lambda} - \frac{l\gamma\delta - d\lambda}{\gamma\delta}$$

$= \frac{\gamma^2 \delta^2 - l^2 \gamma^2 \delta^2 + 2 d l \gamma \delta \lambda - d^2 \lambda^2}{\gamma \delta \times \overline{l \gamma \delta - d \lambda}} = \frac{\gamma^2 \delta^2 \lambda^2 + 2 d l \gamma \delta \lambda - d^2 \lambda^2}{\gamma \delta \times \overline{l \gamma \delta - d \lambda}.} = (d^2 \lambda^2$ being rejected, as incomparably less than the sum of the other terms,) $\lambda . \frac{\gamma \delta \lambda + 2 d l}{l \gamma \delta - d \lambda} = \lambda . \frac{\gamma + 2 r s}{s \gamma - r}$; and, consequently, we have $m . \frac{m-1}{2} . \frac{T^2 - t^2}{\lambda T t} . \dot{L} = m . \frac{m-1}{2} . \frac{29}{100000} . \frac{\gamma + 2 r s}{s \gamma - r}$ minutes. Hence it appears that, in the latitude of Cambridge, when the sun's declination is 2° north, and his distance from the meridian 5°, this error will be equal to $m . \frac{m-1}{2} . \frac{1}{4312}$ minutes of a degree. Let the assumed differ from the true latitude ten minutes, *i. e.* let $m = 10$, then will the error amount to half a second; and, if we suppose $m = 30$, the error will not exceed four seconds.

If, instead of varying the assumed latitude one minute, we vary it n minutes, in calculating the area gc, the above expression will be transformed into $m . \frac{m-1}{2} . \frac{29 . n}{100000} . \frac{\gamma + 2 r s}{s \gamma - r}$, the increment of the latitude being, in this case, $n \dot{L}$. The error will consequently vary as $m . \frac{m-1}{2} . n$, when the declination, latitude, and time of observation are given; and, if we suppose the real to differ from the assumed latitude p minutes, this last expression will become $\frac{p}{n} . \frac{\frac{p}{n} - 1}{2} . n = p . \frac{p-n}{2n}$, which varies as $\frac{p-n}{n}$, when p remains constant; and, of course, the error may be diminished in any proportion by diminishing this quantity $\frac{p-n}{n}$. Now, the increment of the latitude is always $= \lambda T \dot{z}$; and, therefore, since $\dot{z}$ is determined by the process explained above, we have only to ascertain the value of T, in order to approximate nearly to p. But the sine of the azimuth is $= \frac{g\,\delta}{\alpha}$, which is known, and consequently the tangent may likewise be obtained. Having thus discovered a near value of p, or of the error in our first assumed latitude, we are enabled so far to correct it in our second hypothesis, and in a much greater degree to reduce the error in our conclusion.

As it will sometimes, though not often, be necessary to have recourse to this expedient, two short tables might be added, in order to facilitate the operation. The first might contain the log. cosines to every degree from the 15th to the 90th of the quadrant; and be so contrived, as likewise to exhibit the log. sine of any arc, as far as the 75th degree, expressed in minutes and seconds of time; that, by subtracting the log. cosine of the altitude from the log. sine of the hour-angle, (the cosine of declination being considered as equal to radius,) we might obtain the log. sine of the azimuth. This should be made one of the arguments of the second table; the other being the cosine of the latitude to every five degrees of the first sixty of the quadrant; and the table should give us the value of $15 \times \lambda\ T$, which, multiplied into the minutes contained in the error of time, would determine with sufficient exactness the quantity p. We might, indeed, with the same facility, compute the fourth table according to the mean value of the cosine of declination, if it could be supposed that such a degree of precision would ever be required.

Perhaps it would be advisable for the mariner, in order to avoid all distinction of cases, to calculate, in every instance, his incremental area with the lat. varied ten minutes, instead of one; and then he would always be secure of a result sufficiently correct for his purpose. In the case supposed above, if the real were to differ from the assumed latitude 30 minutes, the error in his conclusion would not exceed the fourth part of a second.

We must observe, that if n (the approximate value of p, thus deduced) be the minutes by which the first assumed lat. is varied in our second hypothesis, we ought to multiply the correction, after it is taken from the 2d table, by this quantity; but, as n is always supposed to be a whole number, the additional trouble arising from this process can never be an object of the smallest consideration. The 2d table, however, may be as conveniently applied to the case, where the lat. is varied ten minutes, as where it is varied only one. The logarithms which form the argument are the same in both cases, except that the index in the former is less by unity than in the latter.

Another species of inaccuracy originates in our conceiving the fluxion of the lat. to vary as $T\dot{z}$, instead of $\lambda\, T\dot{z}$; and it may be useful to ascertain how much our conclusion is affected by this circumstance. We suppose, in fact, that the increments of the lat. corresponding to the finite times gf and ge are to each other as the areas gc and gb drawn into the cosine of the first lat.; whereas, in strict propriety, they are to each other as the sum of all the elements of these areas drawn into the cosines of the latitudes belonging to each, *i. e.* (if $ge = m \times gf$) we consider these increments as being in the ratio of $gc \times \lambda : gb \times \lambda$, instead of $gc \times \frac{2\lambda + \dot{\lambda}}{2} : gb \times \frac{2\lambda + m\dot{\lambda}}{2}$. Hence

$$\frac{gb \cdot \frac{2\lambda + m\dot{\lambda}}{2}}{gc \cdot \frac{2\lambda + \dot{\lambda}}{2}} - \frac{gb}{gc} = m \cdot \frac{m-1}{2} \; \frac{\dot{\lambda}}{\lambda^{1}} = m \cdot \frac{m-1}{2} \cdot s\dot{L} \; (\text{rad.} = 1)$$

$= m \cdot \frac{m-1}{2} \cdot \frac{2gs}{100000} =$ the minutes contained in the error proceeding from this cause. When the real latitude, for instance, is 52°, and the assumed 51° 50′, this error will scarcely amount to a single second. But, if either the cosine of the latitude should be so small, or the difference betwixt the supposed and true latitude so great, as to render this error of any importance, we may prevent it by the same means that were recommended in the preceding case. In computing the incremental area *gc*, we must correct our first hypothesis respecting the latitude with the assistance of the third and fourth tables; and, for the reasons assigned above, the cause of this inaccuracy will be removed.

There is still another part of our process, which will sometimes be the source of a small error. We are directed to distribute the whole increment of the time betwixt the two observations, by equalizing the areas corresponding to each in the "*figura tangentium;*" whereas they can only be considered as perfectly equal in the original curve whose ordinates are the tangents of the azimuth. It is supposed, in reality, that *gc* bears to *GC* the same ratio which gc bears to GC; or (which is nearly the same thing) that *gd* is to *GD* as gd to GD: we must therefore estimate the difference betwixt these ratios, and we shall easily deduce the amount of this error. Let us retain the notation employed above, and moreover let Γ represent the cosine, and τ the tangent of the hour-angle at the observation farthest from noon; $\dot{A}$ the difference betwixt the hour-angles, and $\dot{y}$ the excess of γ above Γ: then, from what has already been demonstrated, we have $\frac{gd}{GD} = \frac{l\gamma\delta - d\lambda}{\gamma\delta}$, and $\frac{\mathrm{gd}}{\mathrm{GD}} = \frac{l\Gamma\delta - d\lambda}{\Gamma\delta}$;

consequently, $\frac{gd}{GD} - \frac{\mathrm{gd}}{\mathrm{GD}} = \frac{d\lambda\dot{y}}{\gamma\Gamma\delta}$ ($\dot{d}$ being neglected) will express the part of *BG*, or BG, (when they are supposed to be very small,) by which *bg* exceeds bg. Let this be multiplied into $\frac{GD}{gd}$, and it will equal $\frac{dl\dot{y}}{\Gamma.\overline{l\gamma\delta - d\lambda}} = \frac{r\dot{y}}{\Gamma.\overline{s\gamma - r}} =$ the part of *bg* itself by which it is greater than bg. But our method of equalizing the areas will take from *bg* a portion of this excess bearing to the whole the ratio of $t : t + \tau$; and, consequently, *bg* will be made too small by $\frac{r\dot{y}}{\Gamma.\overline{s\gamma - r}} \times \frac{t}{t+\tau} = \frac{rt\dot{A}}{2.\overline{s\gamma - r}}$ nearly. Hence $\frac{mrt\dot{A}}{2.\overline{s\gamma - r}}$ will express the minutes contained in the error arising from this cause.

The same conclusion may be deduced from the formula $\frac{\dot{\gamma}}{\gamma} = \overline{1 - \frac{r}{s\gamma}} . \frac{\dot{\lambda}}{\lambda^{1}} =$ the area *gb*, if the increment of the time be equal to *ge*. For the same reasons, $\frac{\dot{\Gamma}}{\Gamma} = \overline{1 - \frac{r}{s\Gamma}} . \frac{\dot{\lambda}}{\lambda^{1}}$ = the area gb, if the increment of the time be represented by ge. Hence $gb - \mathrm{gb} = \frac{\dot{\gamma}}{\gamma} - \frac{\dot{\Gamma}}{\Gamma} = \frac{r\dot{y}}{s\gamma\Gamma} \times \frac{\dot{\lambda}}{\lambda^{1}}$, and $\frac{gb - \mathrm{gb}}{gb} \times \frac{t}{t+\tau} = \frac{r\dot{y}}{\Gamma.\overline{s\gamma - r}} \times \frac{t}{t+\tau}$ = that part of the whole area *gb* by which it is made to exceed its proper magnitude; and $\frac{mrt\dot{A}}{2s\gamma - r}$ = the minutes contained in the error which we are investigating. If, for instance, the latitude be 52° 12′, and the declination 2°, both of the same kind, and one of the altitudes be taken at the distance of 5°, the other at the distance of 10°, from the meridian, this error will amount to $\frac{1}{10000}$th part of the whole difference betwixt the real and the assumed latitude. Hence, if this difference be ten minutes, the error in the approximation will not be more than $\frac{1}{17}$th part of a second; and, if the difference be one degree, the error will only be six times greater than in the former case. It will therefore always be inconsiderable, except when the latitude and declination are nearly equal, and on the same side of the equator.

But, if this cause should be likely to produce an effect of some importance, we may prevent it by computing an incremental area with the least altitude likewise, and the latitude increased or diminished by a minute, and then taking a mean betwixt them both for the magnitude of the area *gc*. Thus we shall generally obtain a conclusion sufficiently exact; but, if we are desirous of rendering it perfectly so in every instance, we must divide the difference betwixt the real and apparent intervals, in such a manner that the areas assigned to the two observations may be to each other in the same ratio as their correspondent incremental areas. This division will be readily dispatched by making the difference betwixt the logarithms of the two first terms, in the proportion (taken from the 1st table) equal to the difference betwixt the logarithms of the two last, which are given when either of the formulæ $m^2 - \overline{\frac{r m^3}{s \gamma} \cdot \frac{v s}{m^3}}$, or $\frac{v m}{\lambda} \times \frac{\text{cos. of merid. alt. } (\mu)}{\text{cos. of declin. } (\delta)}$, is employed; but must be found, if the first method of computing the area *gc* be adopted. Suppose that *a* and *b*, representing the logs. of the areas *gb* and gb in the table, appear from inspection to exceed each other nearly as much as the logs. of the areas *gc* and gc; let *c* and *d* be the two next terms in succession, and let *m* and *n* be the logs. of the quantities expressing the incremental areas; then will

$\overline{a \sim b} \sim \overline{c \sim d} : \overline{a \sim b} \sim \overline{m \sim n} :: \overline{a \sim c}$: the quantity to be added to, or subtracted from, *a*, to make it of the required magnitude. We have then only to subtract *m* from this corrected value of *a*, to proceed with the remainder to the second table, and take out the corresponding error in the assumed latitude. This precaution, however, can very seldom be necessary; and, even when it is deemed advisable to adopt it, the division may be performed with so little regard to exactness as to render the process easy and expeditious.

But we are not to conclude that, because this inaccuracy, and also the two first species that were considered, are likely to be introduced, when the latitude and declination are nearly equal and of the same kind, they will therefore unavoidably exist in these circumstances. On the contrary, they may always be prevented, when the weather is favourable, by making the observations within a smaller distance from the meridian. We have only to wait till the increase of the sun's altitude becomes so slow as not to produce a visible separation of the limbs in two or three seconds, and then we may be assured that the azimuth is small, and consequently that none of these errors will be considerable.

I have hitherto supposed that this method is only to be adopted, when the sun, at each observation, is within fifteen degrees of the meridian; or (to speak more accurately) when both the azimuth and the hour-angle are so small that we may consider their tangents as bearing a given ratio to each other; and, indisputably, these limits should never be transgressed, when it can possibly be avoided; for we have seen (page 79th) that, whatever be the method employed, the smaller the hour-angle, the greater is the exactness with which the lat. is determined. Sometimes, however, it will be impossible to make both, or perhaps either of our observations within the distance which I have recommended; but, even in these cases, our rule may be conveniently applied. It has already been demonstrated that we can never be subject to any material error in consequence of the inequality of the areas gb and gb, except when the zenith-distance of the sun, at his meridian altitude, is very small; and, for this case, an effectual remedy has been provided. We need not, therefore, make any farther remarks upon this species of inaccuracy.

But perhaps it will be imagined, that because we still continue to suppose the areas of the "*figura tangentium*" to represent the increments of the latitude, a considerable error will be introduced. We can easily prove, however, that in consequence of the increment of the time being so much diminished by increasing the distance from noon, this error will seldom be of moment enough to claim our attention. Let T be the tangent of the azimuth, and τ the tangent of the hour-angle at the observation farthest from noon; then it follows, from what has been demonstrated before, that the ratio of gb to gc will exceed the ratio of GB to GC, and, consequently, the ratio of gb to gc, (supposed to be of a proper magnitude,) by $m \cdot \frac{m-1}{2} \cdot \frac{T+2rs}{sT-r} \dot{L}$. Hence it is evident that, by diminishing one of these ratios and increasing the other, as we are directed in the present case, till they become equal, we augment the latter by a portion of this difference expressed by $\frac{t}{t+\tau}$, and, on this account, it will be made too great by $m \cdot \frac{m-1}{2} \cdot \frac{t}{t+\tau} \cdot \frac{T+2rs}{sT-r} \cdot \dot{L}$.

But the ratio of gb to gc will itself exceed the ratio of GB to GC, its proper value, by $m . \frac{m-1}{2} . \frac{\gamma + 2rs}{s\gamma - r} . \dot{L}$; and, by the method of equalizing the ratios, it is suffered to retain a portion of this difference expressed by $\frac{\tau}{t+\tau}$; and, therefore, it will be made too great by $m . \frac{m-1}{2} . \frac{\tau}{t+r} . \frac{\gamma + 2rs}{s\gamma - r} . \dot{L}$. The sum of these quantities, $m . \frac{m-1}{2} . \frac{29}{100000} . \frac{1}{t+\tau} . \overline{\frac{t\Gamma + 2rst}{s\Gamma - r} + \frac{\tau\gamma + 2rs\tau}{s\gamma - r}}$ will determine, in minutes of a degree, the amount of the whole error, when the incremental area gc represents one minute of latitude. But if, in our second hypothesis, the latitude be varied n minutes, instead of one, and p be the correction required, the above expression becomes $p . \frac{p-n}{2n} . \frac{29}{100000} . \frac{1}{t+\tau} . \overline{\frac{t\Gamma + 2rst}{s\Gamma - r} + \frac{\tau\gamma + 2rs\tau}{s\gamma - r}}$ minutes. Hence, in the lat. of Cambridge, when the declination is 2° north, and the sun's distance from the meridian at one observation is 5°, and at the other 45°, p being equal to 10, and n equal to unity, the error will amount to little more than half a second.

But, whenever it is judged necessary to guard against this species of error, we need only diminish the value of $\frac{p-n}{n}$ in the last expression; and this will readily be effected by dividing the difference betwixt the real and computed intervals of time, in such a manner that the portions belonging to the two observations may be to each other inversely as the quantities taken out of the fourth table, according to the instructions before delivered. When the portion belonging to the greatest altitude is thus obtained, we can immediately deduce the correction to be made in the first assumed latitude, and then proceed to calculate the incremental areas. Let us suppose, for instance, the difference betwixt the intervals to be q, and the quantities deduced from the 4th table to be a and b; then will $\frac{a b q}{a + b}$ = the minutes by which the former latitude is to be varied.

If, however, there be any considerable uncertainty respecting the latitude, I would recommend the following method of obtaining an approximate value of it, before we begin the process for investigating its real magnitude. This will effectually preclude, in almost every instance, the various errors that have been described. The increment of the altitude is equal to $\lambda S \dot{z}$, and, therefore, when the azimuth is so small that its sine varies nearly as the arc, the whole increase of the altitude, whilst the sun is moving to the meridian, will be $\lambda z \times \frac{S}{2} = \frac{g^2 \lambda \delta}{2 \alpha}$, the sine of the hour-angle being considered as equal to the arc itself. Now the first of the two tables which have just been explained, will enable us to find the value of $\frac{g^2}{\alpha}$, by subtracting the log. cosine of the altitude from twice the log. sine of the hour-angle; and a third table might be added, to furnish us with the whole expression $\frac{g^2 \delta \lambda}{2 \alpha}$, (according to the mean value of δ,) one of its arguments being the quantity $\frac{g^2}{\alpha}$ already determined, and the other the supposed latitude. It might, perhaps, be advisable to add another column to the first of these tables, containing twice the logarithmic sine of the hour-angle, as this would in some measure abridge the operation. We should find it more convenient too, if the last table were to give us the complement of the arc whose value is $\frac{g^2 \delta \lambda}{2 a}$, rather than the arc itself; because it would then only be necessary to subtract the observed altitude from this complement, and we should immediately deduce the zenith distance of the sun, when he had arrived at the meridian. This being ascertained, we should have no farther difficulty in finding the latitude to be adopted in the subsequent computation. The exactness of the conclusion which is derived from this process, will necessarily depend upon the degree of certainty with which the time is given when the observation is made. An error in time may arise, both from an irregularity in the going of the clock and a small inaccuracy in estimating the difference betwixt the longitude of the place where the altitude is now taken, and that where the time was last determined; but these causes, it is evident, can seldom be very considerable. We may generally, I think, be sure of the time within two or three minutes.

The principle upon which this approximation depends will open to us a more compendious way of finding the area *g b*, and which may always be pursued with advantage, when one of the altitudes can be taken at a small distance from the meridian. In the higher latitudes, indeed, the hour-angle may amount to five or six degrees; but, when the latitude and declination approach towards an equality, and are of the same denomination, this angle must be restrained within narrower limits. If, instead of deducing the meridian altitude from the altitude observed, according to the directions before given, we consider it as being actually equal to the meridian altitude, and employ the latitude resulting from this hypothesis in computing the hour-angle, this angle must necessarily be found equal to nothing. We may, therefore, spare ourselves the trouble of performing this part of the operation, and need only calculate the time belonging to the second observation, which is supposed to be made when the sun is at a greater distance from the meridian. The error in the assumed latitude will, in this case, be equal to the whole increase of the altitude betwixt the time of observation and noon; and from this consideration we may be enabled, in any particular instance, to discover at what distance from the meridian this plan may be safely adopted. We can extend it also to any number of observations that happen to be made within the proper limits, by connecting them successively with one on the opposite side of the meridian, and thus determining the areas gb corresponding to each. Having afterwards calculated an incremental area with any of the assumed latitudes varied one minute, we must subtract its logarithm from the logarithmic value of each of the areas gb, and we shall discover, by means of the second table, the correction to be applied to each of the supposed latitudes.

But the method which has just been explained might always be safely adopted, within the limits prescribed to the former rule, if, in computing the incremental area, we were to employ the latitude obtained by means either of the 3d and 4th, or of the 3d and 5th tables. This precaution would entirely prevent the error which might arise from the inequality of the ratios of GC to GB, and gc to gb, when the difference betwixt the real and the assumed latitude was very considerable. The error which is occasioned by the disparity of the hour-angles, might be obviated by the same means that were practised in the first method. There is only one objection, indeed, to this rule's being exclusively adopted, when either of the altitudes is taken within the distance at first recommended. It would be necessary, in that case, to extend the table much farther than we are obliged to do, if we adhere to the former rule; but the labour is so much abridged by this plan, that I am doubtful whether its advantages might not be more than sufficient to compensate this inconvenience. The table should at least be carried to such a length as would enable us to proceed in this manner, whenever an observation was made within five or six degrees of the meridian.

I must not forget to observe, before I conclude the theory, that although I have directed the altitudes to be taken on different sides of the meridian, it is by no means requisite that we should invariably adhere to this precept. We have seen the reason, indeed, why it is expedient, in most instances, to prefer this method, as being generally calculated to produce a much greater degree of exactness in the result. This, however, is not always the case; for, if one of the observations be made beyond the distance originally prescribed, it is of little importance whether the second altitude be taken on the same side of the meridian, or not. But it will sometimes be impossible to make the observations on different sides of noon; and hence it becomes necessary to inquire in what manner the real latitude may be discovered in these circumstances. The clock gives us the interval betwixt the observations equal to $\overline{\mathrm{ae} - ae}$; and by computation we find ag and ag, and thence we deduce $\overline{eg - \mathrm{eg}}$, the difference betwixt the errors in time. Having then assumed, without any regard to accuracy, two portions of time, corresponding to the two observations, whose difference is the same as the difference betwixt the errors before determined, and which are to each other in the inverse ratio of the hour-angles, we must increase or diminish them both equally, till we get the areas in the first table of the same magnitude, and then we may conclude that we have obtained the proper value of each. The directions which have been given for the prevention of errors in the former case, when the altitudes are taken on different sides of the meridian, are very easily accommodated to the present; and it would therefore be superfluous to bestow any farther consideration upon them.

From a review of the inaccuracies to which this method, in particular cases, may be liable, it appears that none of them can ever be of sufficient importance to affect the mariner. If he only computes the time with each of the altitudes and the latitude by account, and an incremental area with the greatest altitude and the former latitude varied ten minutes, the correction will generally be deduced within much less than a second, and, in the most unfavourable circumstances, within a minute, of the truth. But the astronomer, in every instance, even when the latitude and declination are nearly equal and of the same kind, by adopting the precautions which have been recommended, may be assured of a result perfectly exact. If, however, he should entertain any doubts upon this point, he might easily compute a second value of the incremental area with the latitude already determined; and this, it is evident, would necessarily produce a conclusion not less accurate than if it were obtained from the direct method.

The most satisfactory way of proving the utility of this rule, will be to suppose a particular latitude and declination; with these to compute the altitudes, when the sun is at two given distances from the meridian; and thence to deduce the latitude, by an application of our own principles.

EXAMPLE I.

Let the latitude be 54° 27′ 50″ north, and the declin. when the first altit. is taken, 2° 22′ 32″, and, when the second is taken, 2° 21′ 35″, (of the same denomination as the latit.) one observation being made in the morning, when the sun is 5° 3′ 22″, and the other in the afternoon, when he is 10° 1′ 10″, distant from the meridian. The altitude in the former case will be found equal to 37° 44′ 52″; and in the latter to 37° 15′ 20″.

	Observation 1st. Lat. 54° 15′ 0″	Lat. 54° 16′ 0″	Observation 2d. Lat. 54° 15′ 0″
Log. of a =	9,7868838	9,7868838	9,7820217
— d =	8,6175181	8,6175181	8,6146155
— l =	9,9093281	9,9094190	9,9093281
— dl =	18,5268462	18,5269371	18,5239436
— $\frac{am^3}{dl}$ =	21,2600376	21,2599467	21,2580781
— $\left.\frac{am^3}{dl}\right\|^{\frac{1}{2}}$ =	10,6300188	10,6299733	10,6290390
Log. tang. =	10,6177463	10,6176983	10,6167094
2 × log. tang. =	21,2354926	21,2353966	21,2334188
Log. of r =	8,6178915	8,6178915	8,6149839
— s =	10,1427296	10,1429961	10,1427296
Computed log. of γ =	9,9961137	9,9962842	9,9911323
True — =	9,9983068		9,9933253
	Area gb = 21931	9,9961137	Area gb = 21930
		Area gc = 1705	

Hence, $\frac{gb}{gc} = \frac{21931}{1705} = 12' \; 51''$, and latit. = 54° 27′ 51″.

If we employ the formula $\frac{\dot{\gamma}}{\gamma} = m^2 - \overline{\frac{rm^3}{s\gamma}} \cdot \frac{vs}{m^3}$ in computing the area gc, we shall obtain, very nearly, the same result.

Log. of $r =$ 8,6178915

— $s =$ 10,1427296
— $\gamma =$ 9,9961139

— $s\gamma =$ 20,1388435

— $\frac{rm^3}{s\gamma} =$ 18,4790480

— $\overline{\frac{rm^3}{s\gamma}}\Big|^{\frac{1}{2}} =$ 9,2395240
Log. cosine $=$ 9,9933560

2 × log. cos. $=$ 19,9867120
Const. log. $=$ 3,1015
Log. $s =$ 10,1427296

— $gc =$ 3,2309416
— $gb =$ 4,3410188

Difference $=$ 1,1100772, producing in tab. 2d. 12′ 52″
Lat. corrected $=$ 54° 27′ 52″

But there is no occasion to write down the logs. of r, s, and γ, again: we may add together the logs. of s and γ, and subtract their sum from the log. of rm^3 at the same time, as they stand in the former operation, and this will render the latter process extremely short. Let us resume the preceding example, and compute the area gb by the second method, and the area gc by the formula $\overline{m^2 - \frac{rm^3}{s\gamma}} \cdot \frac{vs}{m^3}$.

		Complement of alt. 52° 15′ 8″
Observation 1st.	Observation 2d.	Declin. 2 22 32
Lat. 54° 37′ 40″.	Lat. 54° 37′ 40″.	Lat. 54 37 40
	Log. of a = 9,7820217	
	— d = 8,6146155	
	— l = 9,9113753	
	— $\frac{a m^3}{d l}$ = 21,2560309	
	— $\left.\frac{a m^3}{d l}\right\|^{\frac{1}{2}}$ = 10,6280154	
	Log. tang. = 10,6156259	
	2 × log. tang. = 21,2312518	Log. of $\frac{r m^3}{s \gamma}$ = 18,4711836
	Log. r = 8,6149839	$\left.\frac{r m^3}{s \gamma}\right\|^{\frac{1}{2}}$ = 9,2355918
	— s = 10,1487823	Log. cosine = 9,9934771
Computed log. γ = 10,0000000 - - -	9,9950180	2 × log. cos. = 19,9869542
True - = 9,9983068 - - -	9,9933253	Const. log. = 3,1015
Area $g b$ = 16932	Area $g b$ = 16927	Log. area $g c$ = 3,237
$g b$ + $g b$ = 33859		
Mean value of $g b$ = 16929		
Whose log. (from tab. 1st) - = 4,229		
$g c$ = 3,237		
Difference = 1992	producing from tab. 2d - - - -	9′ 49″
		Lat. corrected = 54° 27′ 51″

EXAMPLE II.

Let the real latitude be 10° 0′ 0″, and the two observations be made on different sides of noon; one when the sun's distance from the meridian is 5° 0′ 0″, and the other when his distance is 10° 0′ 0″; the declin. in the former case being 7° 40′ 40″, and in the latter 7° 39′ 43″, of the same kind as the latitude. Then will the greatest altitude be 84° 32′ 28″, and the least 79° 50′ 48″.

	Observation 1st. Lat. 10° 10′ 0″		Lat. 10° 9′ 0″		Observation 2d. Lat. 10° 10′ 0″
Log. of a =	9,9980259	-	9,9980259	- - -	9,9931449
—— d =	9,1258124	-	9,1258124	- -	9,1249212
—— l =	9,2467746	-	9,2460695	- - -	9,2467746
—— $\frac{a m^3}{d l}$ =	21,6254389	-	21,6261440	- -	21,6214491
—— $\left.\frac{a m^3}{d l}\right\|^{\frac{1}{2}}$ =	10,8127194	-	10,8130720	- -	10,8107245
Log. tang. =	10,8075134	-	10,8078745	- -	10,8054699
2 × log. tang. =	21,6150268	-	21,6157490	- -	21,6109398
Log. of r =	9,1297233	-	9,1297233	- - -	9,1288160
—— s =	9,2536477	-	9,2529200	- -	9,2536477
Computed log. of y =	9,9983978	-	9,9983923	- - -	9,9934035
True —— =	9,9983442	-	-	- -	9,9933515
	Area $g b$ = 536		9,9983978		Area g b = 520
			Area $g c$ = 55		

$\frac{gb + gb}{2} = 528$. Hence, correction $= \frac{528}{55} = 9' \, 34''$, and lat. corrected = 10° 0′ 26″.

The difference, however, betwixt the real and the corrected lat. would only have amounted to 19″, if we had determined the area gb by the 1st table, instead of taking a mean betwixt the two areas for its proper value. This error might have been expected from the near approach of the lat. to the declin. and ought therefore to have been guarded against. It proceeds from the three causes of inaccuracy which, I have shewn, must necessarily be combined in these circumstances. The remedies to be applied are well known. We must find an approximate value of p (page 89) by means of the 3d and 4th tables, and compute an incremental area with the least, as well as with the greatest altitude. If this plan had been pursued, the latitude would have been ascertained within less than a second of the truth. Or, if the weather had been favourable, we might have prevented the error as effectually, by making the observations when the hour-angles were much less, and approached nearer to an equality.

N. B. The log. sines and tangents of the declin. and lat. to be employed in all the operations should be taken out at the same time. But, when the first method of computing the incremental area is adopted, we may avail ourselves, with considerable advantage, of the following expedient. Instead of taking the whole log. sine and tangent of the new lat. take the increments of each, or the difference betwixt their respective values in the two suppositions; and find the increment of the log. tangent corresponding to the increment of the log. secant $\overline{\frac{a m^3}{d l}}\Big|^{\frac{1}{2}}$. This may readily be effected, when the log. tangent is deduced from the log. secant in the former process, by taking the rate of increase belonging to each, and thence inferring by an easy proportion the proper increment of the log. tangent in the latter case. The difference betwixt this increment and the increment of r is the value of the area gc. An instance of the method thus improved shall be given in the next example.

EXAMPLE III.

Let us suppose the true lat. to be 54° 27′ 50″, and two observations to be made in the afternoon—one when the sun's distance from the meridian is 4° 30′ 0″, the other when his distance is 45° 10′ 15″—the declin. in the former case being 8° 7′ 35″, and in the latter 8° 5′ 3″, on the same side of the equator with the latitude. Then will the greatest altitude be found equal to 43° 31′ 19″, and the least to 31° 20′ 25″.

	Observation 1st. Lat 54° 10′ 0″	Observation 2d. Lat. 54° 10′ 0″		Observation 1st. Lat. 54° 11′ 0″	Observation 2d. Lat. 54° 11′ 0″
Log. of a =	9,8379894	9,7161022			
—— d =	9,1503179	9,1480707			
—— l =	9,9088727	9,9088727			
—— $\frac{a\,m^3}{d\,l}$ =	20,7787988	20,6591588	Incr. of log. of l =	912	912
—— $\left.\frac{a\,m^3}{d\,l}\right\|^{\frac{1}{2}}$ =	10,3893994	10,3295794	—— log. secant =	456	456
Log. tang. =	10,3498734	10,2758492	—— tang. =	545	584
2 × log. tang. =	20,6997468	20,5516984	2 × incr. of log. tang. =	1090	1168
Log. of r =	9,1547009	9,1524081			
—— s =	10,1413981	10,1413981	Incr. of log. of s =	2662	2662
Computed log. of y =	9,9958458	9,8455046	Area gc =	1572	gc = 1494
True —— =	9,9986590	9,8481862			
	Area gb = 28132	Area gb = 26816			

Now, by diminishing gb and gb, till the ratio of gc to gc becomes equal to the ratio of gb to gb, we should get $gb = 28126$, and gc = 26735. Hence $\frac{gb}{gc} = \frac{28126}{1572} = 17' \, 53''$ the correction, and the latitude = 54° 27′ 53″.

But the same conclusion would have been obtained independently of the second incremental area, which I have demonstrated above can never be necessary, except when the zenith distance at noon is very small. The error of three seconds arises from a different cause, and might have been entirely excluded with the assistance of the 3d and 4th tables. We should, in fact, deduce from these tables an approximation of 22′; and, if we calculate the area *gc* with the lat. 54° 32′ 0″, it will be found equal to 34762, and consequently $\frac{28172}{36762} = 48'',62$, which, multiplied by 22, produces 17′ 49″,64 for the correction, and the lat. is found = 54° 27′ 49″,64.

It would have been still better, however, to have obtained a near value of the lat. in the first instance, by means of the 3d and 5th tables. The work might have been dispatched in rather less time, and the conclusion would have been as rigidly accurate.

It is obvious that, if the time when the first observation was made could have been ascertained within two or three seconds, the area *gb* might have been immediately found by either of the two methods which have been explained, without the assistance of a second altitude, or of the first table; and the latitude determined with much greater facility, and with sufficient exactness. When the azimuth indeed, as in the present case, does not exceed four or five degrees, an error of a second in time will not produce an error of more than a second in the result; and, as the azimuth decreases, the error in latitude, arising from a given error in time, will be diminished in the same proportion. In general, if $\dot{z}$ be the error in the assumed time, the error in the corrected latitude will be nearly equal to $15\,T\lambda\dot{z}$.

Let us then suppose the time to be given at the first observation, and let us determine the area gb by the second method, and the area gc by the formula $\frac{vm}{\lambda} \cdot \frac{\text{cos. of merid. alt. } (\mu)}{\text{cos. of declin. } (\delta)}$.

Comp. of alt. $= 46° \; 28' \; 41''$
Declin. $= 8 \;\; 7 \;\; 35$

Lat. assumed $= 54 \;\; 36 \;\; 16$

Log. of $v = 3,1015$
—— $\mu = 9,8604043$

—— $mv\mu = 22,9619043$

Computed log. of $\gamma = 10,0000000$
True —— $= 9,9986591$

—— $\delta = 9,9956171$
—— $\lambda = 9,7628420$

Area $gb = 13409$
Log. of area $gb = 4,127$ —— Area $gc = 3,2034452$
———— $gc = 3,203$

Difference $= ,924$ producing from tab. 2d. $8' \; 23''\frac{1}{2}$.
Lat. corrected $= 54° \; 27' \; 52''\frac{1}{2}$.

EXAMPLE IV.

The latitude, which is supposed in the last example to be $54° \; 27' \; 50''$, is that of Ravensworth, a village about five miles to the north of Richmond, in Yorkshire, where I resided some months in the summer of 1797. During that time, I neglected no opportunity of taking the sun's meridian altitude; and, accordingly, the latitude which is there assumed is the mean result of a considerable number of observations. Moreover, on the 8th of September, I took, with particular care, four altitudes, when the sun was near the meridian; and with these I will now calculate the latitude, in order to apply the 1st and 2d tables, and likewise to exemplify the method of combining several observations together. The two first altitudes were taken in the morning, one at 19′ 24″, the other at 17′ 40″ before twelve o'clock, and were respectively 40° 49′ 6″, and 40° 50′ 51″; the two last in the afternoon, one at 17′ 53″, the other at 19′ 44″ past twelve o'clock, and were respectively 40° 49′ 50″, and 40° 48′ 18″. Now the longitude, found by the eclipses of Jupiter's satellites, is nearly 1° 40′ west of Greenwich; and hence the declin. appears to have been, at the two first observations, 5° 26′ 25″, and at the two last, 5° 25′ 47″; the interval betwixt the first and second, and betwixt the third and fourth, being so small that we may consider the declin. as remaining constant during each of these times.

	Observation 1st. Lat. 54° 29′	Observ. 2d. Lat. 54° 29′	Observ. 3d. Lat. 54° 29′	Observ. 4th. Lat. 54° 29′		
Log. of a =	9,8153538	9,8156096	9,8154610	9,8152367		
—— d =	8,9768457	8,9768457	8,9760047	8,9760047		
—— l =	9,9105959	9,9105959	9,9105959	9,9105959		
—— $\frac{a m^3}{d l}$ =	20,9279122	20,9281680	20,9288604	20,9286361	Log. $\frac{r m^3}{s \gamma}$ =	18,8300552
—— $\overline{\frac{a m^3}{d l}}\Big\|^{\frac{1}{2}}$ =	10,4639561	10,4640840	10,4644302	10,4643180	—— $\overline{\frac{r m^3}{s \gamma}}\Big\|^{\frac{1}{2}}$ =	9,4150276
Log. tang. =	10,4366767	10,4368218	10,4372141	10,4370861	Log. cosine =	9,9847972
2 × log. tang. =	20,8733534	20,8736436	20,8744282	20,8741722	2 × l cos. =	19,9695944
Log. of r =	8,9788063	8,9788063	8,9779578	8,9779578	Log. v =	3,1015
—— s =	10,1464648	10,1464648	10,1464648	10,1464648	—— s =	10,1464648
Comput. log. γ =	9,9986245..	9,9989147..	9,9988508..	9,9985948.	Log. $g c$ =	3,218

1st observation.	Time from the argument of 1st table -		18′ 14″, by clock		19′ 24″
4th, ———	———————————		18 26	———	19 44
	Near log. value of *g b* from 1st table	3,296	36 40		39 8
	——— gb ———	3,276			36 40
		2)6,572	Error in time =		2 28
	First log. value of *g b* - -	3,286			
2d observation.	Time from the argument of 1st table -		16′ 12″, by clock		17′ 40″
3d ———	———————————		16 40	———	17 53
			32 52		35 33
	Near log. value of *g b* from 1st table	3,294			32 52
	——— gb ———	3,264			
			Error in time =		2 41
		2)6,558			
	Second log. value of *g b* - -	3,279			
	Mean ——— - -	3,282			
	Log. value of *g c* - - -	3,218			
	Difference	,064	producing in 2d tab.		1′ 10″
			Lat. corrected =		54° 27′ 50″

We now take the same example again, and apply the second method of computing the area *gb*, and the formula $\frac{m v \mu}{\delta \lambda}$ in determining the area *gc*.

Comp. of alt. at 2d observ^n	49° 9′ 9″	Comp. of alt. at 3d observ^n	49° 10′ 10″
Declin. ——	5 26 25	Declin. ——	5 25 47
Lat. =	54 35 34	Lat. =	54 35 57.

	Observations 1st and 3d.	Observations 4th and 2d.
	Lat. 54° 35′ 57″	Lat. 54° 35′ 34″
Log. of a =	9,8153538	9,8152367
—— d =	8,9768457	8,9760047
—— l =	9,9112212	9,9111868
—— $\frac{am^3}{dl}$ =	20,9272869	20,9280452
—— $\left.\frac{am^3}{dl}\right\|^{\frac{1}{2}}$ =	10,4636434	10,4640226
Log. tang. =	10,4363220	10,4367520
2 × log. tang. =	20,8726440	20,8735040
Log. of r =	8,9788063	8,9779578
—— s =	10,1483229	10,1482204
Computed log. of γ =	9,9997732	9,9996822
True —— =	9,9984422	9,9983881
Area gb =	13310	12941
Computed log. of Γ =	10,0000000	10,0000000
True —— =	9,9986765	9,9987084
Area gb =	13235	12916
$\frac{gb + \text{gb}}{2}$ =	13272	12928
Log of $\frac{gb + \text{gb}}{2}$ =	4,1229	4,1115
—— gc =	3,2194	3,2194
Difference =	,9039	,8921
Hence, correction from tab. 2d =	8′ 1″	7′ 48″
Lat. corrected =	54° 27′ 56″	54° 27′ 44″

Mean corrected lat. = 54° 27′ 50″

Log. v =	3,1015
—— μ =	9,8787
$m v \mu$ =	22,9803
—— δ =	9,9980
—— λ =	9,7629
—— Area gc =	3,2194

REMARKS.

1. All the altitudes that are taken on the same side of noon, tend only to correct the error which may be supposed to exist in the greatest of these altitudes, and can have no effect in removing any inaccuracy to which the greatest altitude on the other side of the meridian may be subject. Hence we must take more than one altitude on each side of noon, if we are desirous of obtaining a very exact conclusion.

2. When some of the observations are made in the morning, and others in the afternoon, the smaller the hour-angle, in every instance, the more favourable it will be for our purpose. But, if we cannot procure an altitude on each side of the meridian, we ought to make one observation when the hour-angle is as large as possible, and with this all the rest should be separately combined. We must be cautious, however, not to let the sun be too near the horizon, lest the apparent altitude should be affected by the uncertainty of the refraction.

3. If the clock were to furnish us with the true time, we might combine together any two observations made within the proper limits, without applying to the first table, and deduce a very exact correction. Should there even be a small error in the supposed time, we might still proceed in the same manner, without being liable to any material inaccuracy, provided the difference betwixt the hour-angles was not very considerable. The error, indeed, occasioned by adopting this method of finding the two areas, and taking a mean betwixt them for the value of gb, may easily be determined in any particular case. If t and t' be the respective tangents of the smaller and greater hour-angles, and $\dot{t}$ their difference; $\dot{z}$ the error of the clock in

minutes of time; and $\dot{Z}$ the whole error in the time computed with the greater altitude; then will the error in the result be to the whole correction of the latitude $(\dot{L}) :: \frac{t'\dot{z} - t\dot{z}}{2} : t\dot{z} :: \frac{t\dot{z}}{2} : \frac{t\dot{L}}{15\,T\lambda} :: \frac{t\dot{z}}{2} : \frac{l\gamma\delta - d\lambda}{15\lambda\gamma\delta} \times \dot{L} :: 7{,}5 \times t\dot{z} : \overline{s - \frac{r}{\gamma}}\,\dot{L}$. This error will consequently be equal to $\frac{7{,}5\,t\dot{z}}{s - \frac{r}{\gamma}}$; and hence it appears that, if we had pursued this method in the last example, and there had been an error of a minute in the time given by the clock, there would not have been an error of a single second in the conclusion.

4. If the time were determined by equal altitudes, and one of them were to be employed in computing the area gb, it is manifest that we should entirely exclude the error which has just been considered. It would be necessary, however, in order to correct by the second observation any inaccuracy that may have occurred in reading off at the first, to move the index, and then bring it apparently to the same position again, before we proceeded to take the second altitude.

EXAMPLE V.

On the 30th of January, 1799, the following altitudes of the sun's lower limb were taken in Trinity College, Cambridge; the height of the barometer being 29,6 inches, and of the thermometer 31 degrees.

Observed double alt. by the sextant.	Time by the clock.	Time from noon by the sun.	Log. cos. of the hour-angle.	Lat. from each pair of corresponding altitudes.
39° 35′ 10″	11h 21′ 0″	23′ 40″	9,9976800	52° 12′ 39″
40 7	20 51	20 49	8206	46
44 55	26 49	17 51	8681	45
48 5	29 13	15 27	9012	44
50 26	31 1	13 39	9229	40
52 30	33 0	11 40	9437	41
53 40	34 35	10 5	9579	41
55 19	36 46	7 54	9742	43
56 50	38 56	5 44	9864	38
57 20	41 4	3 36	9946	41
57 55	43 54	0 46	9999	41
57 50	46 40	— 2 0	9983	41
57 28	48 12	3 32	9948	41
57 0	49 59	5 19	9883	38
56 0	51 31	6 51	9806	43
55 15	53 0	8 20	9713	41
53 26	55 2	10 22	9556	41
51 5	57 44	13 4	9295	40
48 38	59 46	15 6	9057	44
45 17	12 2 9	17 29	8735	45
41 50	4 24	19 44	8388	46
35 10	8 20	23 40	7680	39

22)	1102	556	
	39 50	31	
Error of adjust.	- -	48	
2)	39 51	19	
	19 55	39	
Refra. — parall.	2	35	
	19 53	4	
Semi-diameter	16	18	
Assumed merid. alt.	20 9	22	
Mean declin.	17 34	13	
	37 43	35	
Assumed lat.	52 16	25	
Correction from tab. 2d.	3	43	
True lat. -	52 12	42	not less accurate than the mean result from 22 meridian altitudes.

22)	,2034190
	9,9992463
Area gb =	7537
Log. gb =	3,8772
Const. log. + log. rad.	13,1015
Log. cos. merid. alt.	9,9726
	23,0741
— Declin.	9,9793
— Lat.	9,7867
Log. gc =	3,3081
Log. gb — log. gc =	,5691

This process will require very little explanation. The first column contains the observed double altitudes, as they were read off from the sextant; the second contains the corresponding times given by the clock; the third contains the times from noon determined, with sufficient exactness, by taking half the interval betwixt the first and last observations, (in which the altitudes are equal,) and supposing this to be the time elapsed betwixt the first observation and the sun's reaching the meridian. The fourth column is formed of the log. cosines of the hour-angles taken immediately from the argument of the first table; and the last column exhibits the latitudes deduced from every two corresponding altitudes, and is intended to shew the agreement betwixt these results.

It is not necessary that we should employ the tables in this operation: for we may take the log. cosines of the hour-angles from TAYLOR's Logarithms, after the time is reduced into degrees, minutes, and seconds; and, by dividing the area $gb =$ 7537, by the area $gc = 203$, (the natural number belonging to the log. 3,3081,) we shall obtain the correction required.

			0′ 12″	0′ 24″	0′ 36″	0′ 48″	1′ 0″	1′ 12″	1′ 24″	1′ 36″	1′ 48″	2′ 0″	2′ 12″
		9·99	2 . 2	2 . 2	2 . 2	3 . 3	3 . 3	3 . 3	3 . 3	3 . 3	3 . 3	3 . 3	3 . 3
16′	0″	8941	43.42	73.72	908.892	036.014	136.108	217.185	287.249	348.304	401.352	450.395	494.434
—	8	923	43.42	73.72	912.896	039.018	139.112	221.189	290.253	351.308	405.356	453.399	497.438
—	16	905	43.43	74.73	916.899	043.022	143.116	225.192	294.256	354.312	408.360	457.403	501.441
—	24	887	44.43	74.73	919.903	046.025	146.120	228.196	297.260	358.315	411.364	460.407	504.445
—	32	869	44.44	74.73	922.907	050.029	150.123	231.200	301.264	361.319	415.368	463.410	507.449
—	40	850	44.44	75.74	926.910	053.033	153.127	235.203	304.267	365.323	418.371	466.414	510.453
—	48	832	45.44	75.74	929.914	057.036	156.130	238.207	307.271	368.326	421.375	470.418	514.457
—	56	813	45.45	75.74	933.918	060.039	160.134	241.210	311.275	371.330	425.378	473.421	517.460
17	4	795	45.45	76.75	936.921	063.043	163.137	245.214	314.278	374.334	428.382	476.425	520.464
—	12	776	46.45	76.75	940.924	067.046	166.141	248.217	317.282	378.337	431.386	479.429	523.467
—	20	757	46.46	76.75	943.928	070.050	169.145	251.221	321.285	381.341	434.389	483.432	526.471
—	28	737	46.46	77.76	946.931	073.053	173.148	254.225	324.289	384.344	438.393	486.436	530.475
—	36	718	47.46	77.76	949.934	077.057	176.151	257.228	327.292	387.348	441.396	489.439	533.478
—	44	699	47.47	77.76	952.938	080.060	179.155	261.231	330.296	390.351	444.400	492.443	536.482
—	52	679	47.47	78.77	956.941	083.063	182.158	264.235	333.299	394.354	447.403	495.446	539.485
18	0	659	48.47	78.77	959.944	086.067	186.161	267.238	336.303	397.358	450.407	498.450	542.489
—	8	639	48.48	78.77	962.947	090.070	189.165	270.241	339.306	400.361	453.410	501.453	545.492
—	16	619	48.48	79.78	965.951	093.073	192.168	273.245	343.309	403.365	456.413	504.457	548.495
—	24	599	49.48	79.78	968.954	096.077	195.171	276.248	346.312	406.368	459.417	507.460	551.499
—	32	478	49.49	79.78	971.958	099.080	198.174	279.251	349.316	409.371	462.420	510.463	554.502
—	40	558	49.49	80.79	975.960	102.083	201.178	282.254	352.319	412.374	465.423	513.466	557.505
—	48	537	50.49	80.79	978.964	105.086	204.181	286.257	355.322	415.378	468.426	516.470	560.509
—	56	516	50.49	80.79	980.967	108.090	207.184	288.261	358.325	418.381	471.430	519.473	563.512
19	4	495	50.50	80.80	984.970	111.093	210.187	291.264	360.329	421.384	474.433	522.476	566.515
—	12	474	51.50	81.80	987.973	114.096	213.190	294.267	363.332	424.387	477.436	525.479	568.518
—	20	453	51.50	81.80	989.976	117.099	216.193	297.270	366.335	427.390	480.439	528.483	571.522
—	28	431	51.51	81.80	993.979	120.102	219.196	300.273	369.338	430.394	483.442	531.486	574.525
—	36	410	51.51	82.81	996.982	123.105	222.199	303.276	372.341	432.397	486.445	534.489	577.528
—	44	388	52.51	82.81	998.985	125.109	225.202	306.279	375.344	435.400	488.449	536.492	580.531
—	52	366	52.51	82.81	001.988	129.111	228.205	309.282	378.347	438.403	491.452	539.495	583.534
20	0	344	52.52	83.82	004.991	131.114	230.208	312.286	381.350	441.406	494.455	542.498	585.537
—	8	322	53.52	83.82	007.994	134.117	233.212	314.288	383.353	444.409	497.458	545.501	588.540
—	16	300	53.53	83.82	010.997	137.120	236.215	317.291	386.356	446.412	500.461	547.504	591.543
—	24	277	53.53	83.83	012.000	140.123	239.217	320.294	389.359	449.415	502.464	550.507	594.547
—	32	255	53.53	84.83	015.003	142.125	242.220	323.297	392.362	452.418	505.467	553.510	596.549
—	40	232	54.53	84.83	018.006	145.129	244.223	326.300	394.365	455.421	508.470	555.513	599.552
—	48	209	54.54	84.83	021.009	148.131	247.226	328.303	397.368	457.424	510.473	558.516	602.555
—	56	186	54.54	85.84	024.011	151.134	250.229	331.306	400.371	460.427	513.476	561.519	604.558
21	2	163	55.54	85.84	027.015	154.137	252.232	334.309	403.374	463.430	516.479	563.522	607.561
—	12	139	55.54	85.84	029.017	156.140	255.235	336.312	405.376	465.432	519.481	566.525	610.564
—	20	116	55.55	85.85	032.020	159.142	258.237	339.314	408.379	468.435	521.484	569.528	612.567
—	28	092	55.55	86.85	035.022	162.145	261.240	342.317	411.382	471.438	524.487	571.531	615.570
—	36	068	56.55	86.85	037.025	164.148	263.243	344.320	413.385	473.441	526.490	574.534	617.573
—	44	044	56.56	86.85	040.028	167.151	266.246	347.323	416.388	476.444	529.493	576.536	620.576
—	52	020	56.56	86.86	043.031	169.154	268.248	349.326	418.391	478.446	531.496	579.539	622.578
22	0	7996	56.56	87.86	045.033	172.156	271.251	352.328	421.393	481.449	534.498	582.542	625.581
—	8	972	57.56	87.86	048.036	175.159	274.254	355.331	424.396	483.452	537.501	584.545	627.584
—	16	947	57.57	87.86	050.039	177.162	276.256	357.334	426.399	486.455	539.504	587.547	630.587
—	24	922	57.57	88.87	053.041	180.164	279.259	360.336	429.401	489.457	542.507	589.550	632.590
—	32	910	58.57	88.87	056.044	182.167	281.262	362.339	431.404	491.460	544.509	592.553	635.592
—	40	885	58.57	88.87	058.046	185.169	284.264	365.342	434.407	494.463	547.512	594.555	637.595
—	48	860	58.58	88.88	061.049	188.172	286.267	367.344	436.409	496.465	549.515	597.558	640.598
—	56	835	58.58	88.88	063.052	190.175	289.270	370.347	439.412	498.468	551.517	599.561	642.600
23	4	809	58.58	89.88	066.054	193.177	291.272	372.349	441.414	501.471	554.520	602.563	645.603
—	12	784	59.58	89.88	068.057	195.180	294.275	375.352	444.417	503.473	556.522	604.566	647.606
—	20	758	59.59	89.88	071.059	198.182	296.277	377.355	446.420	506.476	559.525	606.569	650.608
—	28	732	59.59	90.89	073.062	200.185	299.280	380.357	448.422	508.478	561.528	609.571	652.611
—	36	706	59.59	90.89	076.064	202.188	301.281	382.360	451.425	511.481	563.530	611.574	654.614
—	44	680	60.59	90.89	078.067	205.190	304.285	385.362	453.427	513.483	566.533	613.576	657.616
—	52	654	60.60	90.90	080.070	207.193	306.288	387.365	455.430	515.486	568.535	616.579	659.619

2′ 12″	2′ 24″	2′ 36″	2′ 48″	3′ 0″	3′ 12″	3′ 24″	3′ 36″	3′ 48″	4′ 0″	4′ 12″
3 · 3	3 · 3	3 · 3	3 · 3	3 · 3	3 · 3	3 · 3	3 · 3	3 · 3	3 · 3	3 · 3
94.434	534.468	571.500	606.530	638.556	669.581	698.605	725.627	751.647	776.666	799.684
97.438	537.472	575.504	609.534	642.561	672.586	701.609	728.631	754.651	779.670	802.688
01.441	541.476	578.508	613.537	645.564	676.590	704.613	732.635	757.655	782.674	806.692
04.445	544.480	581.512	616.541	648.568	679.594	707.617	735.639	761.659	785.678	809.697
07.449	547.484	584.516	619.545	651.572	682.597	711.621	738.643	764.663	788.683	812.701
10.453	551.488	588.520	622.549	655.576	685.601	714.625	741.647	767.667	792.686	815.705
14.457	554.492	591.523	626.553	658.580	688.605	717.628	744.651	770.671	795.690	818.709
17.460	557.495	594.527	629.557	661.584	692.608	720.632	747.654	773.675	798.694	821.712
20.464	560.499	597.531	632.560	664.587	695.613	723.636	750.658	776.679	801.698	824.716
23.467	563.502	601.535	635.564	667.591	698.616	726.640	754.662	779.683	804.702	827.720
26.471	567.506	604.538	638.568	670.595	701.620	729.644	756.666	782.686	807.706	830.724
30.475	570.510	607.542	641.571	673.598	704.624	732.647	760.669	785.690	810.710	833.728
33.478	573.513	610.545	644.575	677.602	707.627	735.651	763.673	788.694	813.713	836.732
36.482	576.517	613.549	647.578	680.606	710.631	738.655	766.677	791.697	816.717	839.735
39.485	579.520	616.552	651.582	683.609	713.635	742.658	769.680	794.701	819.721	842.739
42.489	582.524	619.556	653.585	686.613	716.638	744.662	771.684	797.705	822.724	845.743
45.492	585.527	622.559	656.589	689.616	719.642	747.665	774.688	800.708	825.728	848.746
48.495	588.531	625.563	659.592	692.620	722.645	750.669	777.691	803.712	827.732	851.750
51.499	591.534	628.566	662.596	694.623	725.649	753.672	780.695	806.716	830.735	854.754
54.502	594.537	631.570	665.599	697.627	728.652	756.676	783.698	809.719	833.739	856.757
57.505	597.541	634.573	668.603	700.630	731.656	759.679	786.701	812.722	836.742	859.761
60.509	600.544	637.576	671.606	703.633	733.659	762.683	789.705	814.726	839.746	862.764
63.512	603.547	640.580	674.609	706.637	736.662	765.686	792.708	817.729	842.749	865.768
66.515	606.551	642.583	677.613	709.640	739.666	767.689	794.712	820.733	844.752	868.771
68.518	608.554	645.586	680.616	712.643	742.669	770.693	797.715	823.736	847.756	870.774
71.522	611.557	648.589	682.619	714.647	745.672	773.696	800.718	825.739	850.759	873.778
74.525	614.560	651.593	685.622	717.650	747.676	776.699	803.722	828.743	853.763	876.781
77.528	617.563	654.596	688.626	720.653	750.679	779.703	805.725	831.746	855.766	878.785
80.531	620.567	656.599	691.629	723.656	753.682	781.706	808.728	834.749	858.769	881.788
83.534	623.570	659.602	694.632	726.659	756.685	784.709	811.732	836.753	861.772	884.791
85.537	625.573	662.605	696.635	728.663	758.688	787.712	814.735	839.756	863.776	886.794
88.540	628.576	665.608	699.638	731.666	761.692	789.716	816.738	842.759	866.779	889.798
91.543	631.579	667.611	702.641	734.669	764.695	792.719	819.741	844.762	869.782	892.801
94.547	633.582	670.614	704.644	736.672	766.698	795.722	821.744	847.765	871.785	894.804
96.549	636.585	673.618	707.647	739.675	769. 01	797.725	824.747	849.768	874.788	897.807
99.552	639.588	675.621	710.651	742.678	772. 04	800.728	827.750	852.772	876.792	899.810
02.555	641.591	678.624	712.653	744.681	774. 07	803.731	829.754	855.775	879.795	902.814
04.558	644.594	681.626	715.656	747.684	777. 10	805.734	832.756	857.778	881.798	904.817
07.561	647.597	683.629	718.659	749.687	779. 13	808.737	834.760	860.781	884.801	907.820
10.564	649.600	686.632	720.662	752.690	782. 16	810.740	837.763	862.784	886.804	910.823
12.567	652.603	689.635	723.665	755.693	785. 19	813.743	839.766	865.787	889.807	912.826
15.570	654.606	691.638	725.668	757.696	787. 22	815.746	842.769	867.790	891.810	915.829
17.573	657.608	694.641	728.671	760.699	790. 25	818.749	844.771	870.793	894.813	917.832
20.576	660.611	696. 44	730.674	762.702	792. 28	820.752	847.774	872.796	896.816	919.835
22.578	662.614	699. 47	733.677	765.705	795. 31	823.755	849.777	875.799	899.819	922.838
25.581	665.617	701.650	735.680	767.707	797. 33	825.758	852.780	877.802	901.822	924.841
27.584	667.620	704.652	738.682	770.710	799. 36	828.760	854.783	880.804	904.825	927.844
30.587	670.623	706.655	740.685	772.713	802.739	830.763	857.786	882.807	906.827	929.846
32.590	672.625	709.658	743.688	775.716	804.742	833.766	859.789	884.810	909.830	932.849
35.592	675.628	711.661	745.691	777.719	807.745	835.769	862.792	887.813	911.833	934.852
37.595	677.631	714.663	748.694	779.721	809.747	837.772	864.794	889.816	913.836	936.855
40.598	679.633	716.666	750.696	782.724	812.750	840.774	866.797	892.819	916.839	939.858
42.600	682.636	718.669	752.699	784.727	814.753	842.777	869.800	894.821	918.842	941.861
45.603	684.639	721 672	755.702	787.730	816.756	844.780	871.803	896.824	920.844	943.863
47.606	687.641	723.674	757.704	789.732	819.758	847.783	873.805	899.827	923.847	946.866
50.608	689.644	726.677	760.707	791.735	821.761	849.785	876.808	901.830	925.850	948.869
52.611	691.647	728.679	762.710	794.738	823.764	852.788	878.811	903.832	927.853	950.872
54.614	694.649	730.682	764.712	796 740	826.766	854.791	880.814	906 835	930.855	952.874
57.616	696.652	733.685	767.715	798.743	828.769	856.793	883.816	908.838	932.858	955.877
59.619	698.654	735.687	769.718	801.746	830.772	858.796	885.819	910.840	934.861	957.880

TAB. II.

— 1,176	0′ 4″
— ,875	0 8
— ,699	0 12
— ,577	0 16
— ,477	0 20
— ,399	0 24
— ,331	0 28
— ,273	0 32
— ,222	0 36
— ,176	0 40
— ,134	0 44
— ,097	0 48
— ,062	0 52
— ,030	0 56
,000	1 0
,028	1 4
,054	1 8
,079	1 12
,103	1 16
,125	1 20
,146	1 24
,166	1 28
,186	1 32
,204	1 36
,222	1 40
,239	1 44
,255	1 48
,271	1 52
,286	1 56
,301	2 0

TAB. III.

h. ′.	° ′	Log. of g or α.	Log. of g^2.
0 1	89 45	7,64	15,28
0 2	89 30	7,94	15,88
0 3	89 15	8,12	16,24
0 4	89 0	8,24	16,48
0 5	88 45	8,34	16,68
0 6	88 30	8,42	16,84
0 7	88 15	8,48	16,96
0 8	88 0	8,54	17,08
0 9	87 45	8,59	17,18
0 10	87 30	8,64	17,28
0 11	87 15	8,68	17,36
0 12	87 0	8,72	17,44
0 13	86 45	8,75	17,50
0 14	86 30	8,78	17,56
0 15	86 15	8,81	17,62
0 16	86 0	8,84	17,68
0 17	85 45	8,87	17,74
0 18	85 30	8,89	17,78
0 19	85 15	8,92	17,84
0 20	85 0	8,94	17,88
0 22	84 30	8,98	17,96
0 24	84 0	9,02	18,04
0 26	83 30	9,05	18,10
0 28	83 0	9,08	18,16
0 30	82 30	9,11	18,22
0 32	82 0	9,14	18,28
0 34	81 30	9,17	18,34

TAB. IV.

	10°	20°	30°	35°	40°	45°	50°	55°	60°
8,64	,64	,61	,57	,54	,50	,46	,42	,38	,33
8,68	,70	,67	,69	,59	,55	,51	,46	,41	,36
8,71	,77	,74	,68	,64	,60	,55	,50	,45	,39
8,75	,84	,80	,74	,70	,65	,60	,55	,49	,43
8,78	,90	,86	,79	,75	,70	,65	,59	,53	,46
8,81	,97	,92	,85	,81	,75	,69	,63	,56	,49
8,84	1, 0	,98	,91	,86	,80	,74	,67	,60	,52
8,87	1, 1	1, 0	,96	,91	,85	,79	,71	,64	,56
8,89	1, 2	1, 1	1, 0	,97	,90	,83	,76	,68	,59
8,92	1, 2	1, 2	1, 1	1, 0	,95	,88	,80	,71	,62
8,94	1, 3	1, 2	1, 1	1, 1	1, 0	,93	,84	,75	,66
8,98	1, 4	1, 3	1, 2	1, 2	1, 1	1, 0	,93	,83	,72
9,02	1, 5	1, 5	1, 4	1, 3	1, 2	1, 1	1, 0	,90	,79
9,05	1, 7	1, 6	1, 5	1, 4	1, 3	1, 2	1, 1	,98	,86
9,09	1, 8	1, 7	1, 6	1, 5	1, 4	1, 3	1, 2	1, 1	,92

TAB. V.

	10°	20°	30°	35°	40°	45°	50°	55°	60°
	89°	89°	89°	89°	89°	89°	89°	89°	89°
6,463	59′ 30″	59′ 32″	59′ 34″	59′ 36″	59′ 38″	59′ 40″	59′ 42″	59′ 44″	59′ 46″
6,765	— 0	— 4	— 8	— 11	— 14	— 18	— 21	— 26	— 30
6,941	58 30	58 35	58 42	58 46	58 51	58 56	— 2	— 8	— 15
7,066	— 0	— 7	— 14	— 22	— 28	— 35	58 43	58 51	58 59
7,163	57 30	57 38	57 49	57 57	— 5	— 14	— 23	— 33	— 45
7,242	— 0	— 11	— 24	— 33	57 42	57 53	— 4	— 17	— 30
7,309	56 30	56 42	56 58	— 8	— 19	— 32	57 45	57 59	— 12
7,367	— 0	— 14	— 32	56 43	56 56	— 10	— 25	— 42	— 0
7,418	55 30	55 46	— 6	— 19	— 34	56 50	— 6	— 25	57 45
7,464	— 0	— 18	55 40	55 54	— 10	— 28	56 47	— 7	— 30
7,505	54 30	54 49	— 14	— 39	55 47	— 7	— 28	56 48	— 15
7,543	— 0	— 21	54 48	— 5	— 14	55 45	— 8	— 33	56 59
7,578	53 30	53 50	— 22	54 40	— 1	— 14	55 49	— 16	— 45
7,610	— 0	— 25	53 56	— 16	54 38	— 3	— 30	55 59	— 29
7,640	52 30	52 57	— 30	53 51	— 15	54 42	— 11	— 41	— 15

EXPLANATION OF THE TABLES.

Tab. I. This table contains two arguments. One is the distance of the sun from the meridian, which is expressed in hours, minutes, and seconds; and also in terms of the log. cosine of the hour-angle. This argument is placed in the first column. The second stands at the top of the table, and consists of the minutes and seconds contained in the error of time. The table itself exhibits the logarithm of the area *g b* corresponding to any given error at any particular observation. But it was necessary to give the log. value of this area in both directions from the tangent *g d*, i. e. both when the computed is greater, and when it is less, than the real interval of time. These two values are included in the same column, but are separated from each other by a point. The first quantity expresses the log. area in the latter, the second expresses it in the former case. Thus, for instance, in the fifth column we have 136.108, and we are to understand that the log. of the area *gb* is equal to 136, when the computed time is too small; and equal to 108, when the computed time is too great.

The argument at the top is carried to every twelve seconds of time, or every three minutes of a great circle; and should be continued as far as four or five minutes of time for the first twelve or fourteen degrees from the meridian; as far as three minutes for the next ten degrees; two minutes for the next ten degrees; and so on, (the time being diminished as the tangent of the hour-angle increases,) as far as sixty or sixty-five degrees, beyond which the table need not be extended. The argument in the first column should be carried to every eight seconds of time for the first twelve or fourteen degrees; and to every twenty seconds for the rest of the table.

Tab. II. This table consists of two columns. The first contains the differences betwixt the logs. of the area *gb* and *gc*, when the latit. is varied one minute in calculating the incremental area. These differences are negative as far as the fourteenth, because so far the area *gc* exceeds the area *gb*. In the second column are exhibited the errors in the assumed latit. corresponding to these differences.

Tab. III. This table is composed of four columns. The first contains the computed distance in time from noon; the second the altitude of the sun; the third the log. sine of the hour-angle, or the log. cosine of the altitude, expressed in the first column; and the fourth twice the logarithm in the former column, being intended to save us the trouble of multiplying by 2, when we are finding the logarithm of $\frac{g^2}{a}$.

Tab. IV. In this table there are two arguments. One of them occupies the first column, and consists of the log. sines of the azimuth, which are found by subtracting the log. cosine of the altitude from the log. sine of the hour-angle, as they stand in the preceding table. The other argument is placed at the top of the columns, and is formed of the assumed latitude. The table is designed to exhibit the product of the tangent of the azimuth multiplied into the cosine of the latit. and into 15; so that, by taking out this quantity, and multiplying it into the minutes contained in the error of time, we may at once determine how much the assumed latit. is to be varied in computing the incremental area.

Tab. V. There are two arguments in this table likewise. That which stands in the first column is the log. value of $\frac{g^2}{a}$ obtained from the second table by subtracting the log. cosine of the altitude from twice the log. sine of the hour-angle. The second argument, which is placed above the columns, is the supposed latitude. The table itself contains the complement of the meridian altitude at the place of observation.

TABLE I.

To face page 116.

		0' 12"	0' 24"	0' 36"	0' 48"	1' 0"	1' 12"	1' 24"	1' 36"	1' 48"	2' 0"	2' 12"	2' 24"	2' 36"	2' 48"	3' 0"	3' 12"	3' 24"	3' 36"	3' 48"	4' 0"	4' 12"
	9.99	2.2	2.2	2.2	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3
16' 0"	8941	43.42	73.72	908.892	036.014	136.108	217.185	287.249	348.304	401.352	450.395	494.434	534.468	571.500	606.530	638.556	669.581	698.605	725.627	751.647	776.666	799.684
— 8	923	43.42	73.72	912.896	039.018	139.112	221.189	290.253	351.308	405.356	453.399	497.438	537.472	575.504	609.534	642.561	672.586	701.609	728.631	754.651	779.670	802.688
— 16	905	43.43	74.73	916.899	043.022	143.116	225.192	294.256	354.312	408.360	457.403	501.441	541.476	578.508	613.537	645.564	676.590	704.613	732.635	757.655	782.674	806.692
— 24	887	44.43	74.73	919.903	046.025	146.120	228.196	297.260	358.315	411.364	460.407	504.445	544.480	581.512	616.541	648.568	679.594	707.617	735.639	761.659	785.678	809.697
— 32	869	44.44	74.73	922.907	050.029	150.123	231.200	301.264	361.319	415.368	463.410	507.449	547.484	584.516	619.545	651.572	682.597	711.621	738.643	764.663	788.683	812.701
— 40	850	44.44	75.74	926.910	053.033	153.127	235.203	304.267	365.323	418.371	466.414	510.453	551.488	588.520	622.549	655.576	685.601	714.625	741.647	767.667	792.686	815.705
— 48	832	45.44	75.74	929.914	057.036	156.130	238.207	307.271	368.326	421.375	470.418	514.457	554.492	591.523	626.553	658.580	688.605	717.628	744.651	770.671	795.690	818.709
— 56	813	45.45	75.74	933.918	060.039	160.134	241.210	311.275	371.330	425.378	473.421	517.460	557.495	594.527	629.557	661.584	692.608	720.632	747.654	773.675	798.694	821.712
17 4	795	45.45	76.75	936.921	063.043	163.137	245.214	314.278	374.334	428.382	476.425	520.464	560.499	597.531	632.560	664.587	695.613	723.636	750.658	776.679	801.698	824.716
— 12	776	46.45	76.75	940.924	067.046	166.141	248.217	317.282	378.337	431.386	479.429	523.467	563.502	601.535	635.564	667.591	698.616	726.640	754.662	779.683	804.702	827.720
— 20	757	46.46	76.75	943.928	070.050	169.145	251.221	321.285	381.341	434.389	483.432	526.471	567.506	604.538	638.568	670.595	701.620	729.644	756.666	782.686	807.706	830.724
— 28	737	46.46	77.76	946.931	073.053	173.148	254.225	324.289	384.344	438.393	486.436	530.475	570.510	607.542	641.571	673.598	704.624	732.647	760.669	785.690	810.710	833.728
— 36	718	47.46	77.76	949.934	077.057	176.151	257.228	327.292	387.348	441.396	489.439	533.478	573.513	610.545	644.575	677.602	707.627	735.651	763.673	788.694	813.713	836.732
— 44	699	47.47	77.76	952.938	080.060	179.155	261.231	330.296	390.351	444.400	492.443	536.482	576.517	613.549	647.578	680.606	710.631	738.655	766.677	791.697	816.717	839.735
— 52	679	47.47	78.77	956.941	083.063	182.158	264.235	333.299	394.354	447.403	495.446	539.485	579.520	616.552	651.582	683.609	713.635	742.658	769.680	794.701	819.721	842.739
18 0	659	48.47	78.77	959.944	086.067	186.1[illegible]1	267.238	336.303	397.358	450.407	498.450	542.489	582.524	619.556	653.585	686.613	716.638	744.662	771.684	797.705	822.724	845.743
— 8	639	48.48	78.77	962.947	090.070	189.165	270.241	339.306	400.361	453.410	501.453	545.492	585.527	622.559	656.589	689.616	719.642	747.665	774.688	800.708	825.728	848.746
— 16	619	48.48	79.78	965.951	093.073	192.168	273.245	343.309	403.365	456.413	504.457	548.495	588.531	625.563	659.592	692.620	722.645	750.669	777.691	803.712	827.732	851.750
— 24	599	49.48	79.78	968.954	096.077	195.171	276.248	346.312	406.368	459.417	507.460	551.499	591.534	628.566	662.596	694.623	725.649	753.672	780.695	806.716	830.735	854.754
— 32	478	49.49	79.78	971.958	099.080	198.174	279.251	349.316	409.371	462.420	510.463	554.502	594.537	631.570	665.599	697.627	728.652	756.676	783.698	809.719	833.739	856.757
— 40	558	49.49	80.79	975.960	102.083	201.178	282.254	352.319	412.374	465.423	513.466	557.505	597.541	634.573	668.603	700.630	731.656	759.679	786.701	812.722	836.742	859.761
— 48	537	50.49	80.79	978.964	105.086	204.181	286.257	355.322	415.378	468.426	516.470	560.509	600.544	637.576	671.606	703.633	733.659	762.683	789.705	814.726	839.746	862.764
— 56	516	50.49	80.79	980.967	108.090	207.184	288.261	358.325	418.381	471.430	519.473	563.512	603.547	640.580	674.609	706.637	736.662	765.686	792.708	817.729	842.749	865.768
19 4	495	50.50	80.80	984.970	111.093	210.187	291.264	360.329	421.384	474.433	522.476	566.515	606.551	642.583	677.613	709.640	739.666	767.689	794.712	820.733	844.752	868.771
— 12	474	51.50	81.80	987.973	114.096	213.190	294.267	363.332	424.387	477.436	525.479	568.518	608.554	645.586	680.616	712.643	742.669	770.693	797.715	823.736	847.756	870.774
— 20	453	51.50	81.80	989.976	117.099	216.193	297.270	366.335	427.390	480.439	528.483	571.522	611.557	648.589	682.619	714.647	745.672	773.696	800.718	825.739	850.759	873.778
— 28	431	51.51	81.80	993.979	120.102	219.196	300.273	369.338	430.394	483.442	531.486	574.525	614.560	651.593	685.622	717.650	747.676	776.699	803.722	828.743	853.763	876.781
— 36	410	51.51	82.81	996.982	123.105	222.199	303.276	372.341	432.397	486.445	534.489	577.528	617.563	654.596	688.626	720.653	750.679	779.703	805.725	831.746	855.766	878.785
— 44	388	52.51	82.81	998.985	125.109	225.202	306.279	375.344	435.400	488.449	536.492	580.531	620.567	656.599	691.629	723.656	753.682	781.706	808.728	834.749	858.769	881.788
— 52	366	52.51	82.81	001.988	129.111	228.205	309.282	378.347	438.403	491.452	539.495	583.534	623.570	659.602	694.632	726.659	756.685	784.709	811.732	836.753	861.772	884.791
20 0	344	52.52	83.82	004.991	131.114	230.208	312.286	381.350	441.406	494.455	542.498	585.537	625.573	662.605	696.635	728.663	758.688	787.712	814.735	839.756	863.776	886.794
— 8	322	53.52	83.82	007.994	134.117	233.212	314.288	383.353	444.409	497.458	545.501	588.540	628.576	665.608	699.638	731.666	761.692	789.716	816.738	842.759	866.779	889.798
— 16	300	53.53	83.82	010.997	137.120	236.215	317.291	386.356	446.412	500.461	547.504	591.543	631.579	667.611	702.641	734.669	764.695	792.719	819.741	844.762	869.782	892.801
— 24	277	53.53	83.83	012.000	140.123	239.217	320.294	389.359	449.415	502.464	550.507	594.547	633.582	670.614	704.644	736.672	766.698	795.722	821.744	847.765	871.785	894.804
— 32	255	53.53	84.83	015.003	142.125	242.220	323.297	392.362	452.418	505.467	553.510	596.549	636.585	673.618	707.647	739.675	769. 01	797.725	824.747	849.768	874.788	897.807
— 40	232	54.53	84.83	018.006	145.129	244.223	326.300	394.365	455.421	508.470	555.513	599.552	639.588	675.621	710.651	742.678	772. 04	800.728	827.750	852.772	876.792	899.810
— 48	209	54.54	84.83	021.009	148.131	247.226	328.303	397.368	457.424	510.473	558.516	602.555	641.591	678.624	712.653	744.681	774. 07	803.731	829.754	855.775	879.795	902.814
— 56	186	54.54	85.84	024.011	151.134	250.229	331.306	400.371	460.427	513.476	561.519	604.558	644.594	681.626	715.656	747.684	777. 10	805.734	832.756	857.778	881.798	904.817
21 2	163	55.54	85.84	027.015	154.137	252.232	334.309	403.374	463.430	516.479	563.522	607.561	647.597	683.629	718.659	749.687	779. 13	808.737	834.760	860.781	884.801	907.820
— 12	139	55.54	85.84	029.017	156.140	255.235	336.312	405.376	465.432	519.481	566.525	610.564	649.600	686.632	720.662	752.690	782. 16	810.740	837.763	862.784	886.804	910.823
— 20	116	55.55	85.85	032.020	159.142	258.237	339.314	408.379	468.435	521.484	569.528	612.567	652.603	689.635	723.665	755.693	785. 19	813.743	839.766	865.787	889.807	912.826
— 28	092	55.55	86.85	035.022	162.145	261.240	342.317	411.382	471.438	524.487	571.531	615.570	654.606	691.638	725.668	757.696	787. 22	815.746	842.769	867.790	891.810	915.829
— 36	068	56.55	86.85	037.025	164.148	263.243	344.320	413.385	473.441	526.490	574.534	617.573	657.608	694.641	728.671	760.699	790. 25	818.749	844.771	870.793	894.813	917.832
— 44	044	56.56	86.85	040.028	167.151	266.246	347.323	416.388	476.444	529.493	576.536	620.576	660.611	696. 44	730.674	762.702	792. 28	820.752	847.774	872.796	896.816	919.835
— 52	020	56.56	86.86	043.031	169.154	268.248	349.326	418.391	478.446	531.496	579.539	622.578	662.614	699. 47	733.677	765.705	795. 31	823.755	849.777	875.799	899.819	922.838
22 0	7996	56.56	87.86	045.033	172.156	271.251	352.328	421.393	481.449	534.498	582.542	625.581	665.617	701.650	735.680	767.707	797. 33	825.758	852.780	877.802	901.822	924.841
— 8	972	57.56	87.86	048.036	175.159	274.254	355.331	424.396	483.452	537.501	584.545	627.584	667.620	704.652	738.682	770.710	799. 36	828.760	854.783	880.804	904.825	927.844
— 16	947	57.57	87.86	050.039	177.162	276.256	357.334	426.399	486.455	539.504	587.547	630.587	670.623	706.655	740.685	772.713	802.739	830.763	857.786	882.807	906.827	929.846
— 24	922	57.57	88.87	053.041	180.164	279.259	360.336	429.401	489.457	542.507	589.550	632.590	672.625	709.658	743.688	775.716	804.742	833.766	859.789	884.810	909.830	932.849
— 32	910	58.57	88.87	056.044	182.167	281.262	362.339	431.404	491.460	544.509	592.553	635.592	675.628	711.661	745.691	777.719	807.745	835.769	862.792	887.813	911.833	934.852
— 40	885	58.57	88.87	058.046	185.169	284.264	365.342	434.407	494.463	547.512	594.555	637.595	677.631	714.663	748.694	779.721	809.747	837.772	864.794	889.816	913.836	936.855
— 48	860	58.58	88.88	061.049	188.172	286.267	367.344	436.409	496.465	549.515	597.558	640.598	679.633	716.666	750.696	782.724	812.750	840.774	866.797	892.819	916.839	939.858
— 56	835	58.58	88.88	063.052	190.175	289.270	370.347	439.412	498.468	551.517	599.561	642.600	682.636	718.669	752.699	784.727	814.753	842.777	869.800	894.821	918.842	941.861
23 4	809	58.58	89.88	066.054	193.177	291.272	372.349	441.414	501.471	554.520	602.563	645.603	684.639	721.672	755.702	787.730	816.756	844.780	871.803	896.824	920.844	943.863
— 12	784	59.58	89.88	068.057	195.180	294.275	375.352	444.417	503.473	556.522	604.566	647.606	687.641	723.674	757.704	789.732	819.758	847.783	873.805	899.827	923.847	946.866
— 20	758	59.59	89.88	071.059	198.182	296.277	377.355	446.420	506.476	559.525	606.569	650.608	689.644	726.677	760.707	791.735	821.761	849.785	876.808	901.830	925.850	948.869
— 28	732	59.59	90.89	073.062	200.185	299.280	380.357	448.422	508.478	561.528	609.571	652.611	691.647	728.679	762.710	794.738	823.764	852.788	878.811	903.832	927.853	950.872
— 36	706	59.59	90.89	076.064	202.188	301.281	382.360	451.425	511.481	563.530	611.574	654.614	694.649	730.682	764.712	796.740	826.766	854.791	880.814	906.835	930.855	952.874
— 44	680	60.59	90.89	078.067	205.190	304.285	385.362	453.427	513.483	566.533	613.576	657.616	696.652	733.685	767.715	798.743	828.769	856.793	883.816	908.838	932.858	955.877
— 52	654	60.60	90.90	080.070	207.193	306.288	387.365	455.430	515.486	568.535	616.579	659.619	698.654	735.687	769.718	801.746	830.772	858.796	885.819	910.840	934.861	957.880